Higher Orders and Multiplicities with Herwig++

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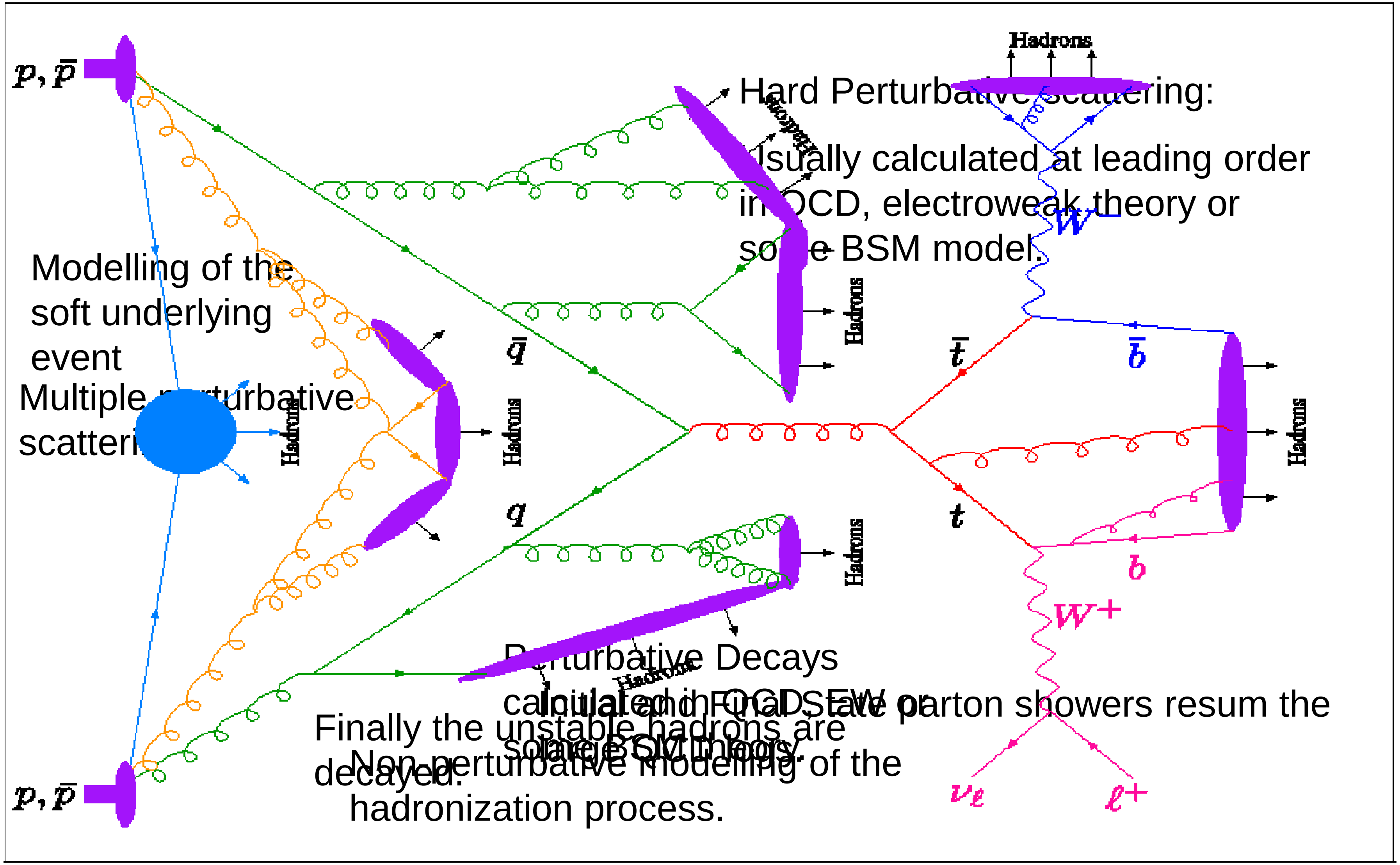
Summary

- Introduction
- Basics of Event Generation
- Hard Radiation
- Next-to-leading Order
- High Multiplicity Jet Production
- Photon Production
- Conclusions

Introduction

- Monte Carlo event generators are essential for experimental particle physics.
- They are used for:
  - Comparison of experimental results with theoretical predictions;
  - Studies for future experiments.
- Often these programs are ignored by theorists and treated as black boxes by experimentalists.
- It is important to understand the assumptions and approximations involved in these simulations.

A Monte Carlo Event



Hard Perturbative scattering:

Usually calculated at leading order in QCD, electroweak theory or some BSM model.

Modelling of the soft underlying event

Multiple perturbative scattering.

Perturbative Decays calculated in QCD, EW or some BSM theory.

Initial and Final State parton showers resum the large QCD logs.

Finally the unstable hadrons are decayed.

Non-perturbative modelling of the hadronization process.

Parton Shower

- The parton shower is designed to simulate the bulk of the QCD radiation which is either soft or collinear.
- This takes into account the dominant QCD corrections however:
  - the leading order normalisation is retained;
  - it fails to describe additional hard QCD radiation.

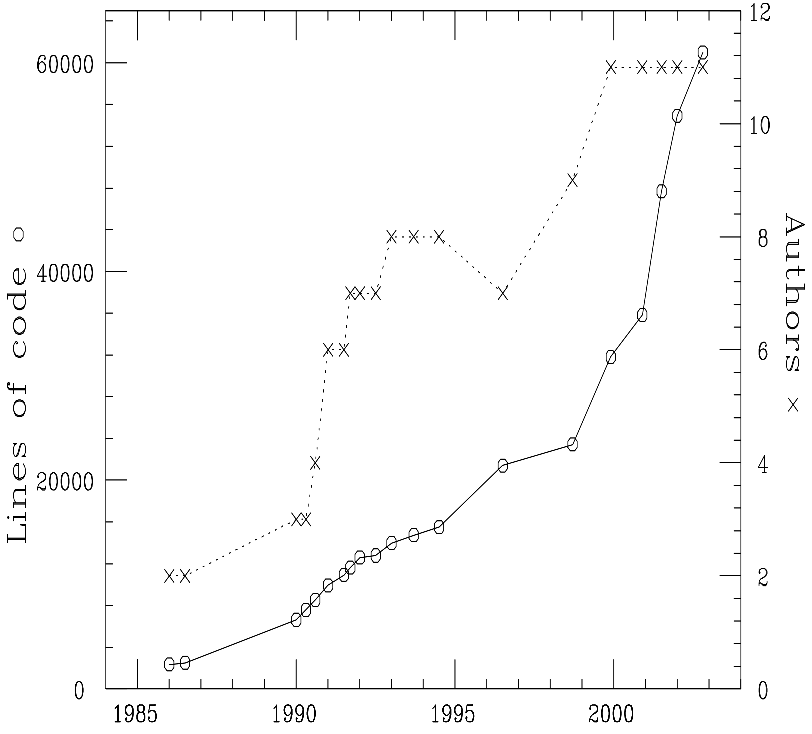
Parton Shower

- At the LHC however we will need both:
  - the improved normalisation and scale uncertainty that next-to-leading order cross sections provide;
  - a good description of the production of final states with large numbers of jets which are important backgrounds in the search for new physics.
- In this talk I will describe work I have been involved with as part of the Herwig++ project to produce better simulations including these effects.

Herwig++

M. Baehr, S. Gieseke, M. Gigg, D. Grellscheid, K. Hamilton, S. Plaetzer, PR, M.H. Seymour, J. Tully

- The Herwig++ project aims to produce a new Monte Carlo event generator based on the physics philosophy and models of the successful HERWIG program.
- Not just a rewrite we aim to make a number of improvements to the simulation physics.



Parton Shower

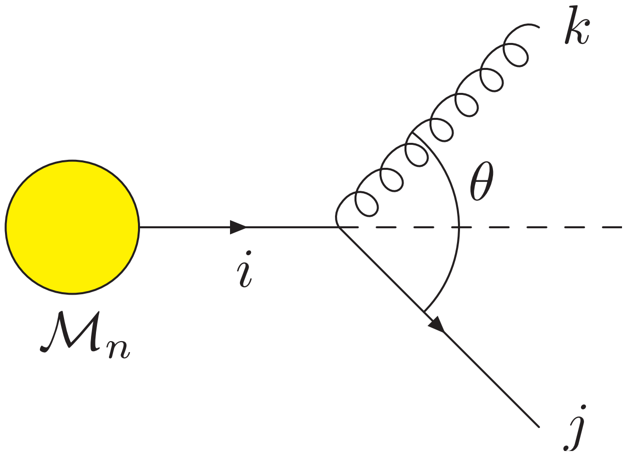
- Before we can go on and consider improvements to the simulation of QCD radiation we first need to review how Monte Carlo generators simulate QCD radiation.
- All parton shower simulations rely on the factorization of the cross section for QCD emission in the soft and collinear limits.

Collinear Singularities

- In the collinear limit the cross section for a process factorizes

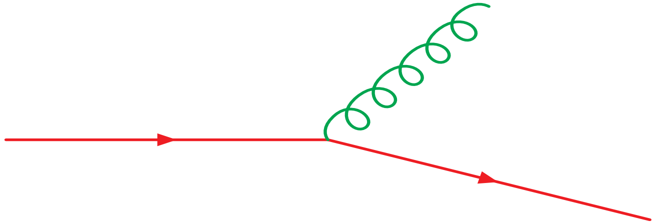
$$d\sigma_{n+1} = d\sigma_n \frac{d\theta^2}{\theta^2} dz \frac{\alpha_s}{2\pi} P_{ji}(z)$$

  - $P_{ji}(z)$ is the DGLAP splitting function.
- The splitting function only depends on the spin and flavours of the particles



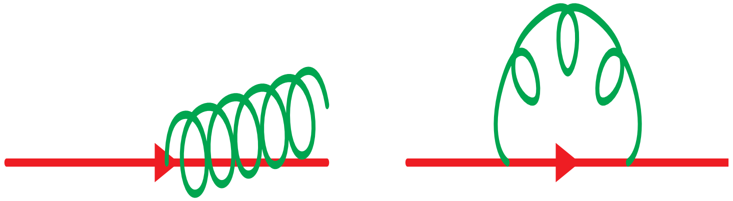
Collinear Singularities

- This expression is singular as θ→0.
- What is a parton? (or what is the difference between a collinear pair and a parton)
- Introduce a resolution criterion, e.g. $k_T > Q_0$
- Combine the virtual corrections and unresolvable emission



Resolvable Emission

Finite



Unresolvable Emission

Finite

- Unitarity: Unresolved + Resolved =1

Monte Carlo Procedure

- Using this approach we can exponentiate the real emission piece.

$$\text{Unresolved} = 1 - \text{Resolved} = 1 - \int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz \frac{\alpha_s}{2\pi} P_{ji}(z)$$

$$= \exp\left(-\int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz \frac{\alpha_s}{2\pi} P_{ji}(z) \right)$$

- This gives the Sudakov form factor which is the probability of evolving between two scales and emitting no resolvable radiation.
- More strictly it is the probability of evolving from a high scale to the cut-off with no resolvable emission.

Numerical Procedure

- Start with a parton at a high virtuality, *Q*, typical of the hard collision.
- Work out the scale of the next branching by generating a random number $R \in [0,1]$ and solving

$$R = \Delta(Q^2, q^2)$$

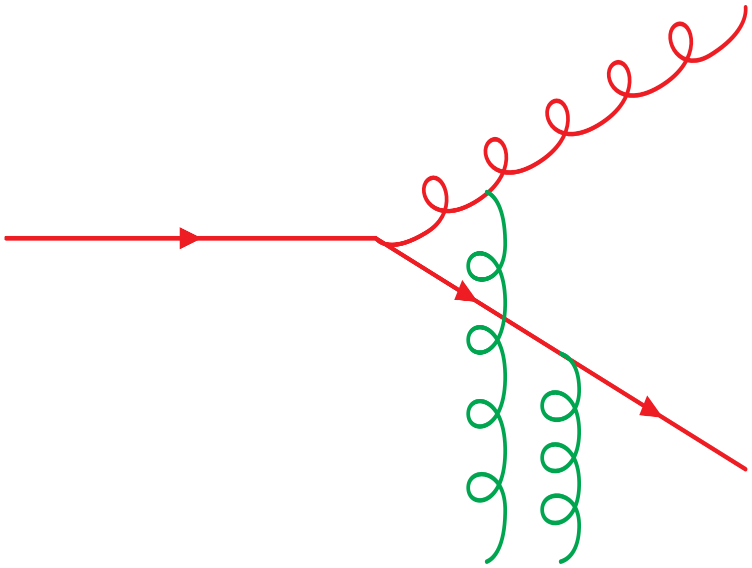
  where q is the scale of the next branching
- If there's no solution for q bigger than the cut-off stop.
- Otherwise workout the type of branching, energy fraction *z* and azimuthal angle, ϕ.
- Repeat the process for the partons produced in the branching.

Monte Carlo Procedure

- The key difference between the different Monte Carlo simulations is in the choice of the evolution variable.
- Evolution Scale
  - Virtuality, q^2
  - Transverse Momentum, k_T.
  - Angle, θ.
  - ….
- Energy fraction, z
  - Energy fraction
  - Light-cone momentum fraction
  - ….
- All are the same in the collinear limit.

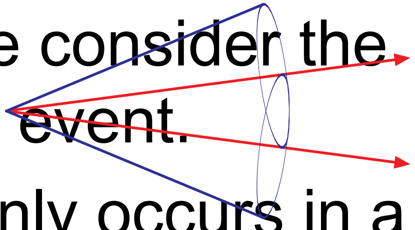
Soft Emission

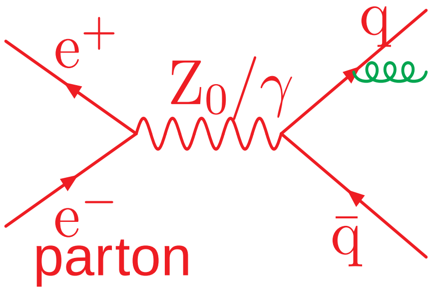
- We have only considered collinear emission. What about soft emission?
- In the soft limit the matrix element factorizes but at the amplitude level.
- Soft gluons come from all over the event.
- There is quantum interference between them.
- Does this spoil the parton shower picture?



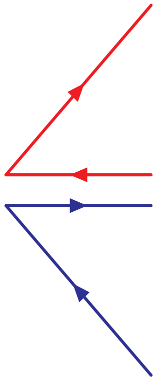
Angular Ordering

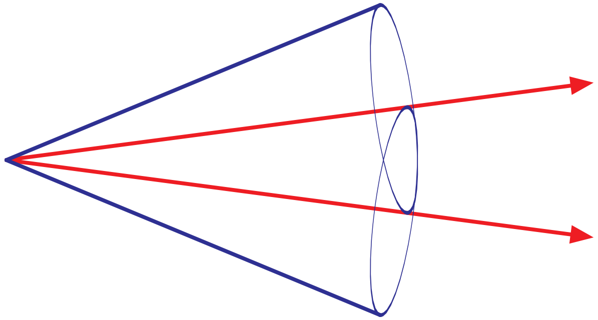
- There is a remarkable result that if we take the large number of colours limit much of the interference is destructive.
- In particular if we consider the colour flow in an event.
- QCD radiation only occurs in a cone up to the direction of the colour partner.
- The best choice of evolution variable is therefore an angular one.





Colour Flow





Parton Shower

- At the end of the parton shower we have a set of parameters $\tilde{q}, \; z, \; \phi$ for each emission in the shower.
- In Herwig++ these map into the momenta of the partons using the Sudakov decomposition

$$q = \alpha p + \beta n + q_{\perp}$$

where

p is the momentum of the particle which started the shower,

n is a reference vector,

the transverse momentum $q_{\perp}$ is calculated using $\tilde{q}$ and ϕ

α is calculated using z

β is fixed by requiring the on-shell partons at the end of the shower

Parton Shower

- For a single branching $p \to q_1 + q_2$

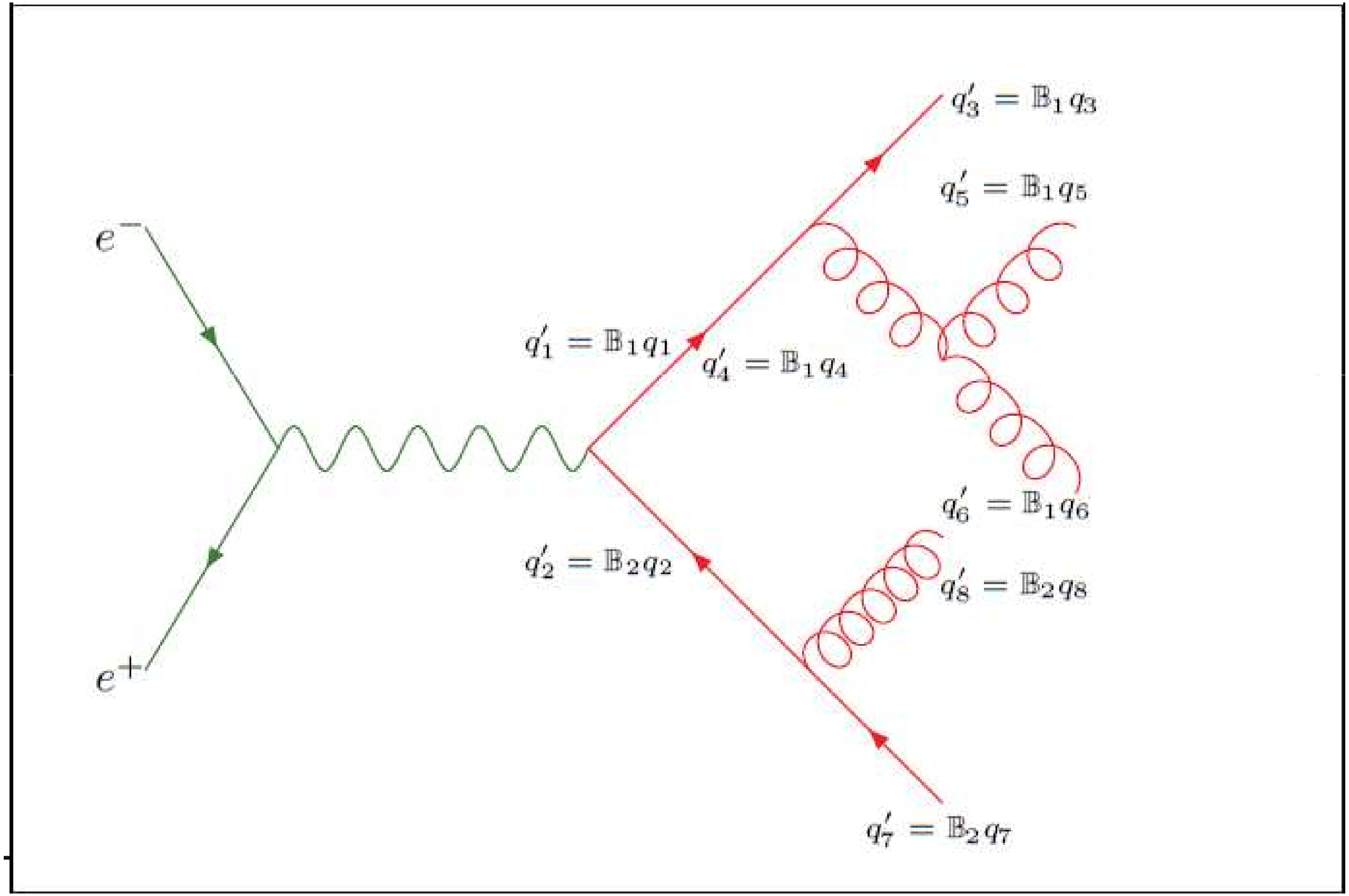
$$\alpha_1 = z$$

$$\alpha_2 = 1 - z$$

$$-q_\perp^2 = p_T^2 = \frac{\tilde{q}^2}{z^2(1-z)^2}$$

- This leaves the partons we started with off-shell and so we need to apply a boost to the momenta of each of the new jets to ensure global energy and momentum conservation while preserving the masses of the jets.

Parton Shower Evolution



Hard Radiation

- In the angular ordered parton shower the hardest emission in p_T is not the emission with the largest emission scale.
- In fact the hardest emission is often proceeded by softer wide angle emissions.
- This is a problem if we want to use a fixed order matrix element to describe the hardest emission.
- However work by Nason JHEP 0411:040,2004 showed how to do this.

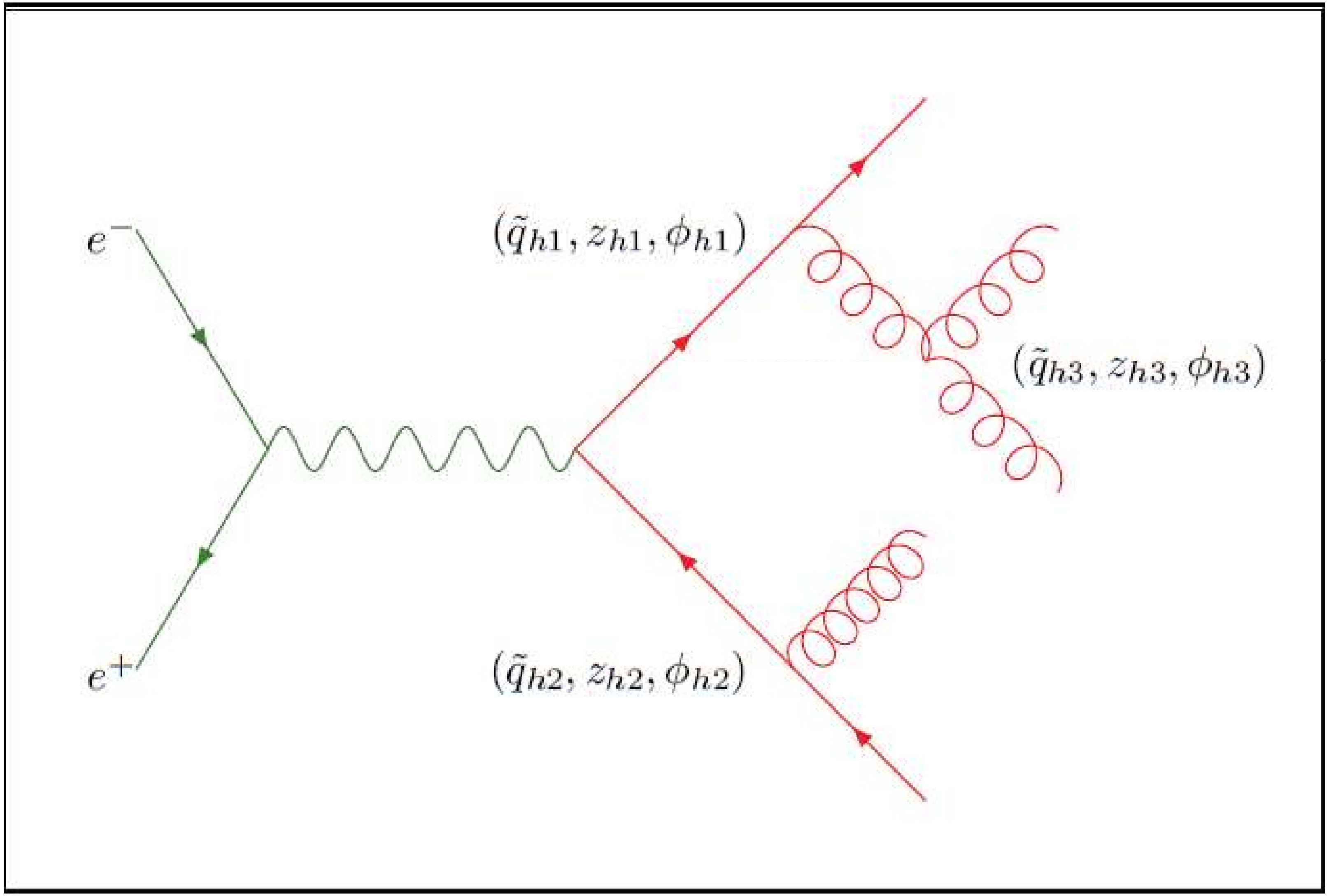
Hard Radiation

- Nason showed that the angular ordered parton shower could be decomposed into:
  - the hardest emission;
  - a truncated shower describing soft wide angle emission at higher evolution scales than the hardest emission;
  - vetoed showers from the partons constrained to only generate emissions softer than the hardest one.
- This allows the hardest emission to be generated separately.
- Problem is how to do it in practice.

Hard Radiation

- In Herwig++ we use a simple approach to implement this procedure:
  - the momenta including the hard emission are generated;
  - the inverse of the boosts applied in the shower to conserve energy and momentum are applied;
  - the Sudakov decomposition is used to find the shower evolution variables for the hardest emission.
- Once this is done we can generate the full shower in one step.

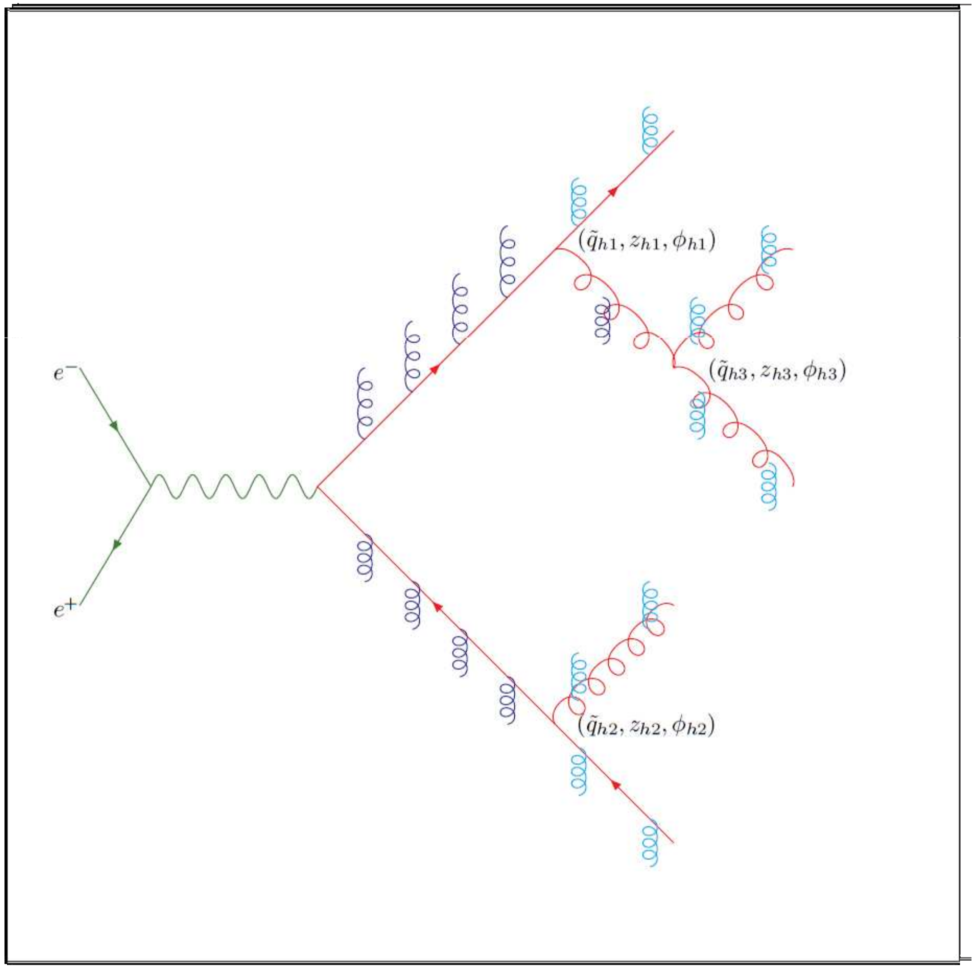
Hard Radiation



Hard Radiation in the Shower

- Start at the normal shower starting scale.
- Generate emissions as normal but forbid emissions which violate the conditions of the truncated shower.
- When the evolution scale falls below the scale of the hardest emission we insert a branching with the right shower variables.
- Generate the rest of the shower as normal vetoing emission with p_T above that of the hardest emission.

Shower including Hard Radiation



Hard Radiation in the Shower

- This relatively simple approach allows us to improve the simulation of additional hard radiation in the Herwig++ angular ordered parton shower.
- I will now go on and describe how we have done this for a range of processes:
  - Drell-Yan at NLO;
  - Higgs Production at NLO;
  - DIS and VBF at NLO;
  - $e^+e^-\rightarrow$hadrons at leading order;
  - Drell-Yan at leading order.

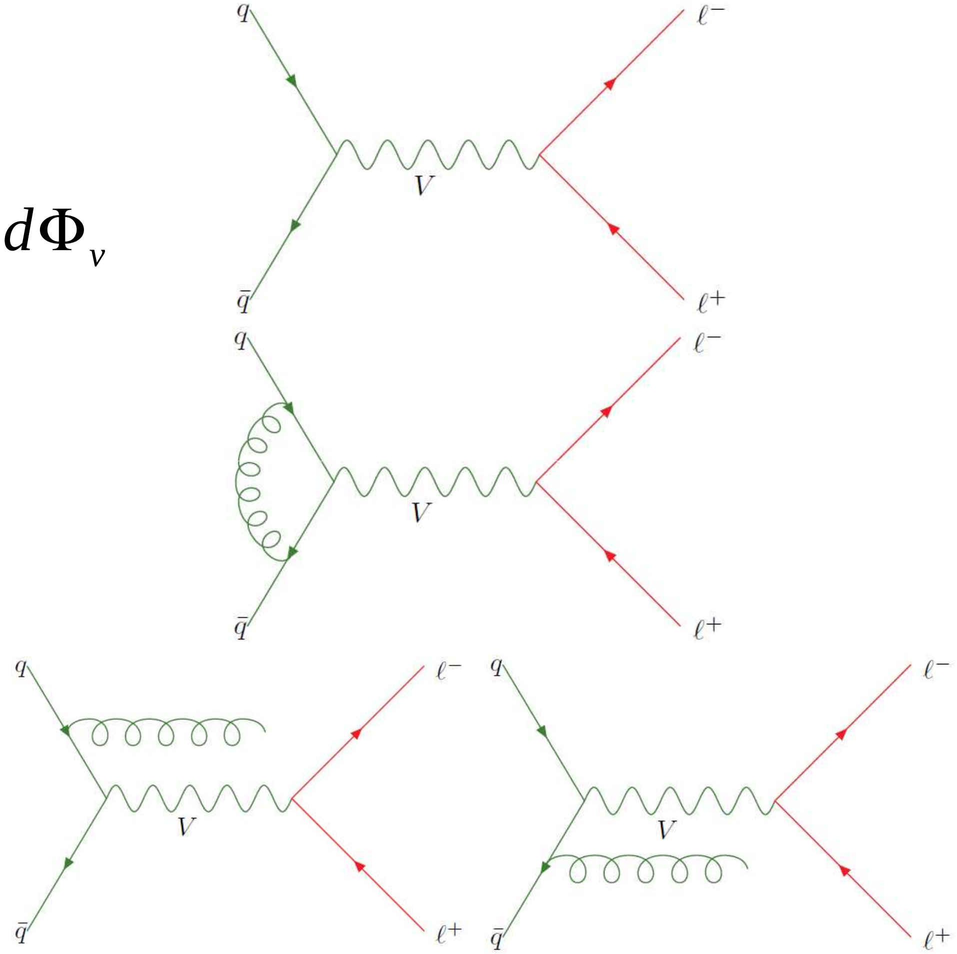
NLO Simulations

- The NLO cross section is

$$
\begin{aligned}
d\sigma = {} & B(v)d\Phi_v + (V(v) + C(v,r)d\Phi_r)d\Phi_v \\
& + (R(v,r) - C(v,r))d\Phi_v d\Phi_r
\end{aligned}
$$

  putting all the pieces together the answer is finite.

- The problem for many years was how to use this to produce a Monte Carlo simulation.

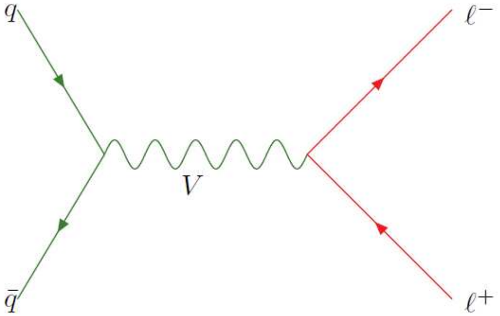


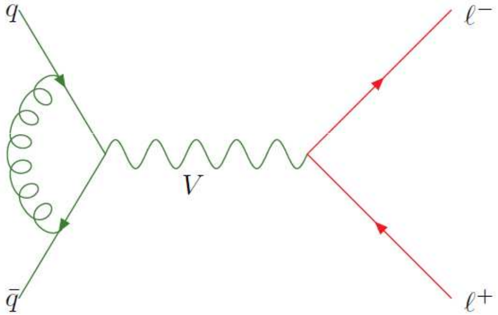
NLO Simulations

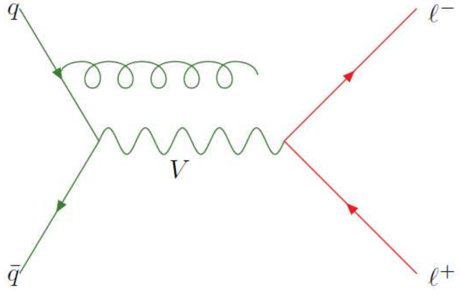
- NLO simulations rearrange the NLO cross section formula.
- Either choose C to be the shower approximation

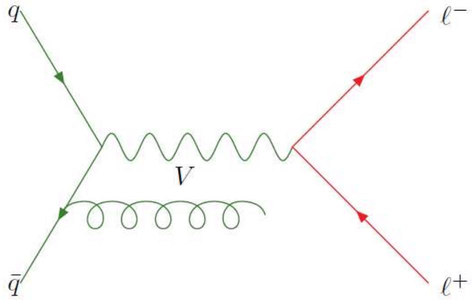
$$d\sigma = B(v)d\Phi_v + (V(v) + C_{\text{shower}}(v,r)d\Phi_r)d\Phi_v$$
$$+ (R(v,r) - C_{\text{shower}}(v,r))d\Phi_v d\Phi_r$$

MC@NLO (Frixione, Webber)









NLO Simulations

- Or a more complex arrangement POWHEG(Nason)

$$d\sigma = \bar{B}(v)d\Phi_v \left[\Delta_R^{(\mathrm{NLO})}(0) + \Delta_R^{(\mathrm{NLO})}(p_\mathrm{T})\frac{R(v,r)}{B(v)}d\Phi_r\right]$$

where

$$\bar{B}(v) = B(v) + V(v) + \int \left(R(v,r) - C(v,r)\right) d\Phi_r$$

$$\Delta_R^{(\mathrm{NLO})}(p_\mathrm{T}) = e^{-\int d\Phi_r \frac{R(v,r)}{B(v)}\theta(k_\mathrm{T}(v,r)-p_\mathrm{T})}$$

- Looks more complicated but has the advantage that it is independent of the shower and only generates positive weights.

POWHEG

- We can think of the cross section

$$d\sigma = \bar{B}(v)d\Phi_v \left[\Delta_R^{(\mathrm{NLO})}(0) + \Delta_R^{(\mathrm{NLO})}(p_{\mathrm{T}})\frac{R(v,r)}{B(v)}d\Phi_r\right]$$

  where

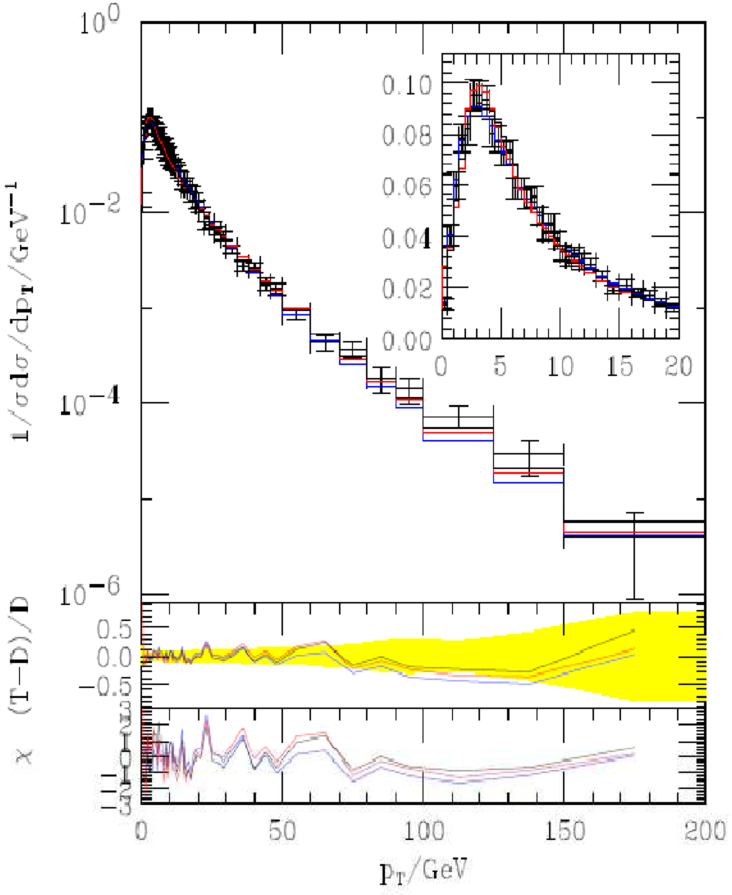
$$\bar{B}(v) = B(v) + V(v) + \int (R(v,r) - C(v,r))\, d\Phi_r$$

  in two parts:
  - first generate the momenta of the partons in the Born process with NLO accuracy according to $\bar{B}(v)$;
  - Generate the additional hard emission using

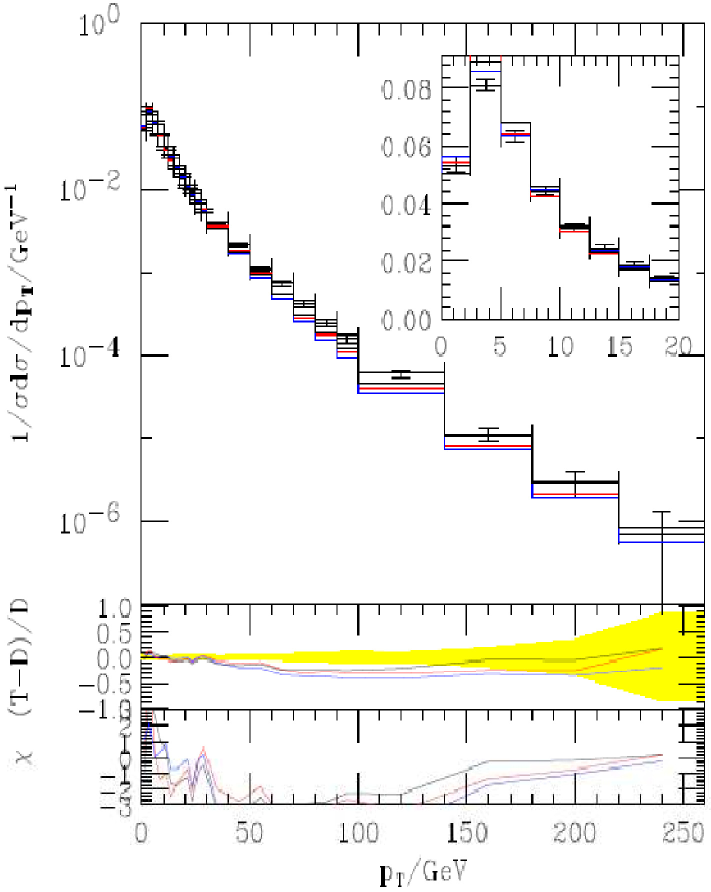
$$\Delta_R^{(\mathrm{NLO})}(p_{\mathrm{T}}) = e^{-\int d\Phi_r \frac{R(v,r)}{B(v)}\theta(k_{\mathrm{T}}(v,r)-p_{\mathrm{T}})}$$

- The procedure I outlined before can then be used to generate the full parton shower.

POWHEG method for Drell-Yan



CDF Run I Z p_T



D0 Run II Z p_T

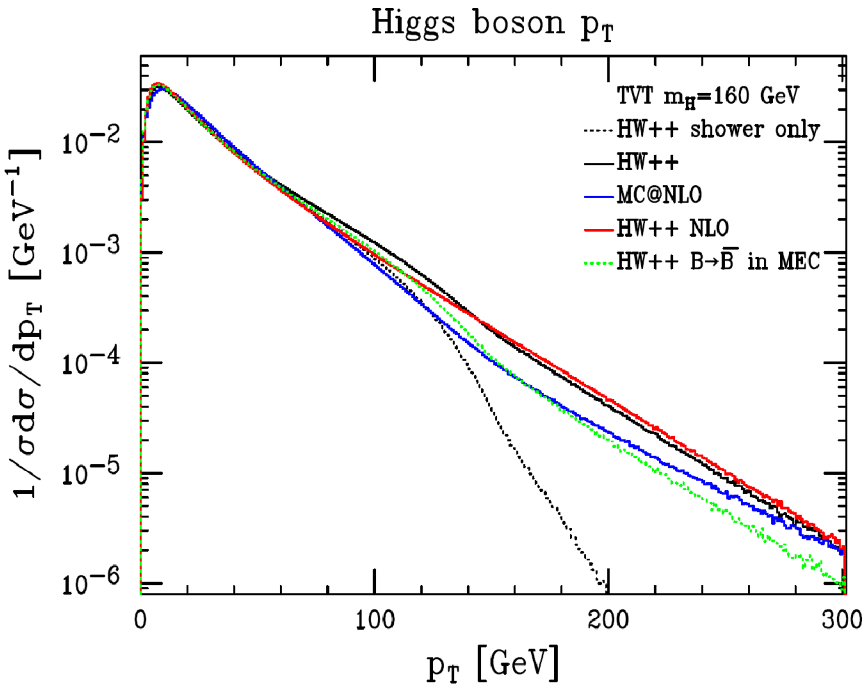
Herwig++

POWHEG

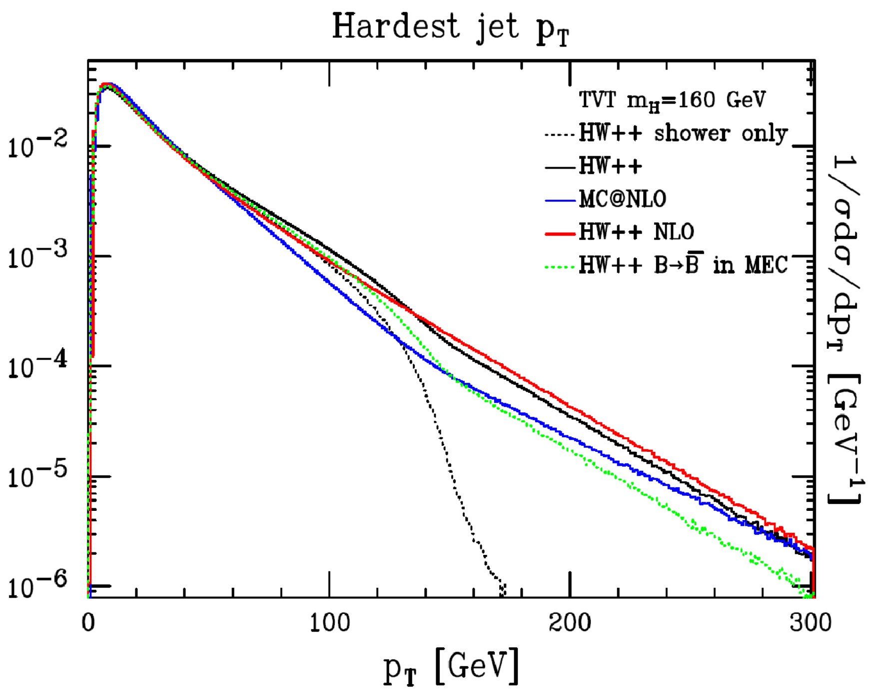
MC@NLO

JHEP 0810:015,2008 Hamilton, PR, Tully

POWHEG Method for gg→H



Tevatron



LHC

JHEP 0904:116,2009 Hamilton, PR, Tully

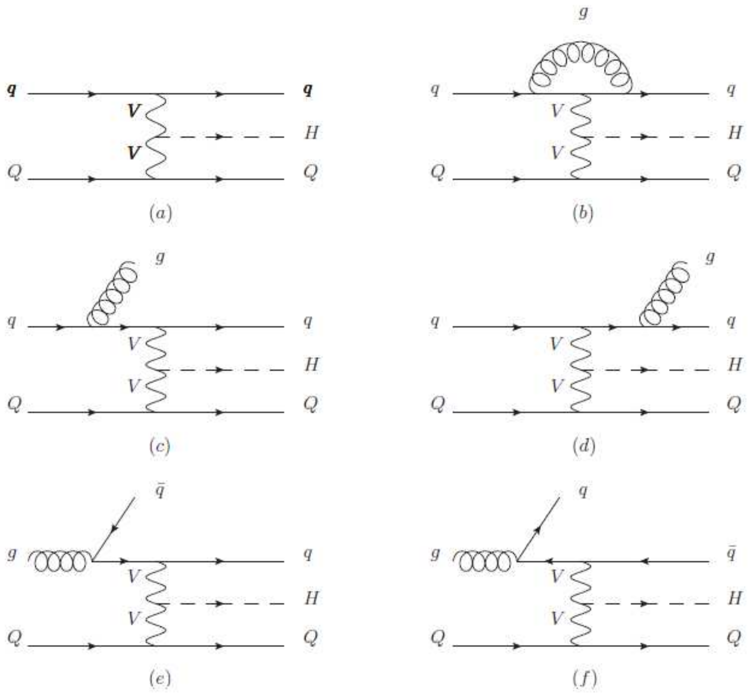
Deep Inelastic Scattering

- In Herwig++ the final boosts are applied to conserve the invariant mass of colour singlet systems, i.e.:
  - the final-state quark-antiquark pair in e+e-;
  - the gauge boson in Drell-Yan processes.
- In processes where there's a t-channel vector boson, e.g.:
  - Deep Inelastic scattering (DIS);
  - Higgs production via vector boson fusion,

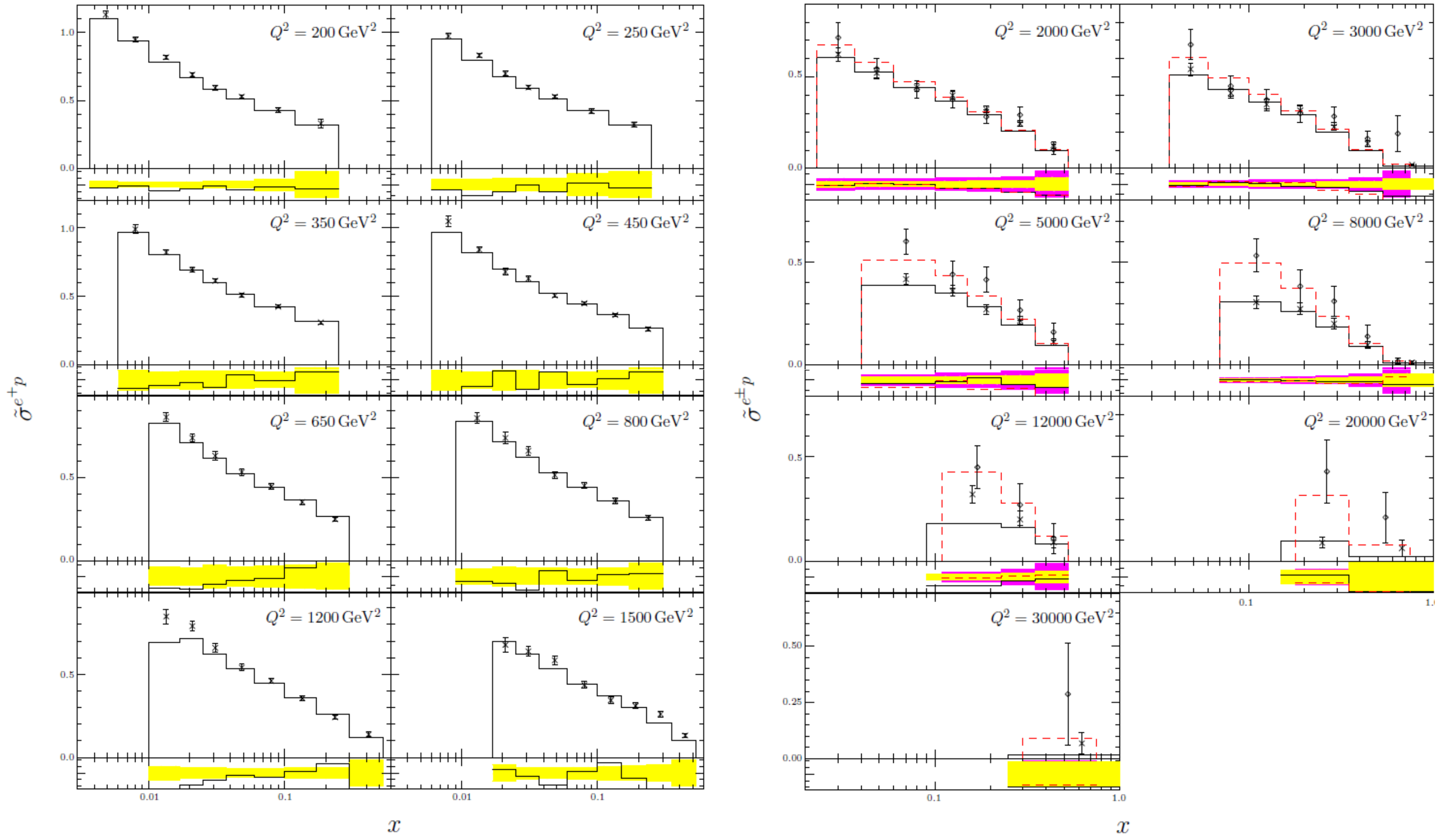
we preserve the momentum of this boson.

Deep Inelastic Scattering

- The QCD corrections to the two processes are also similar.
- DIS is important is its own right for validating and tuning new event generators.
- Also a useful testing ground for our treatment of the VBF process which is important for Higgs boson searches at the LHC.

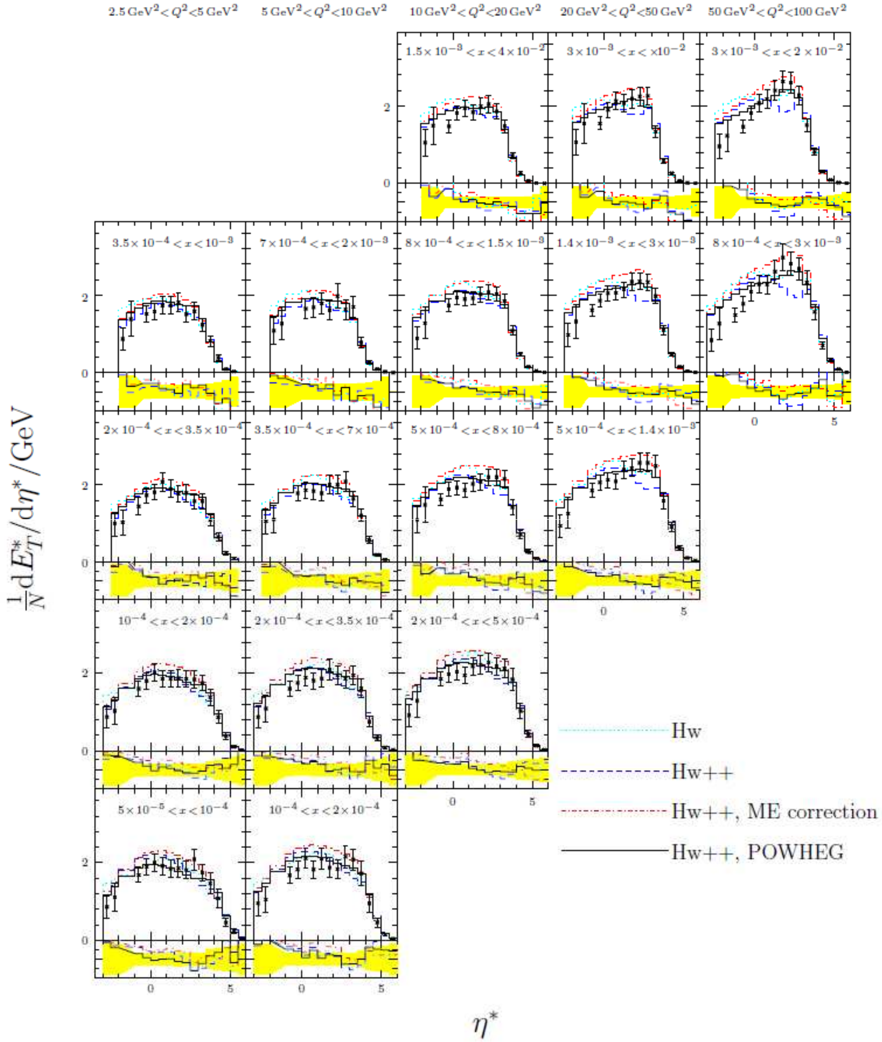


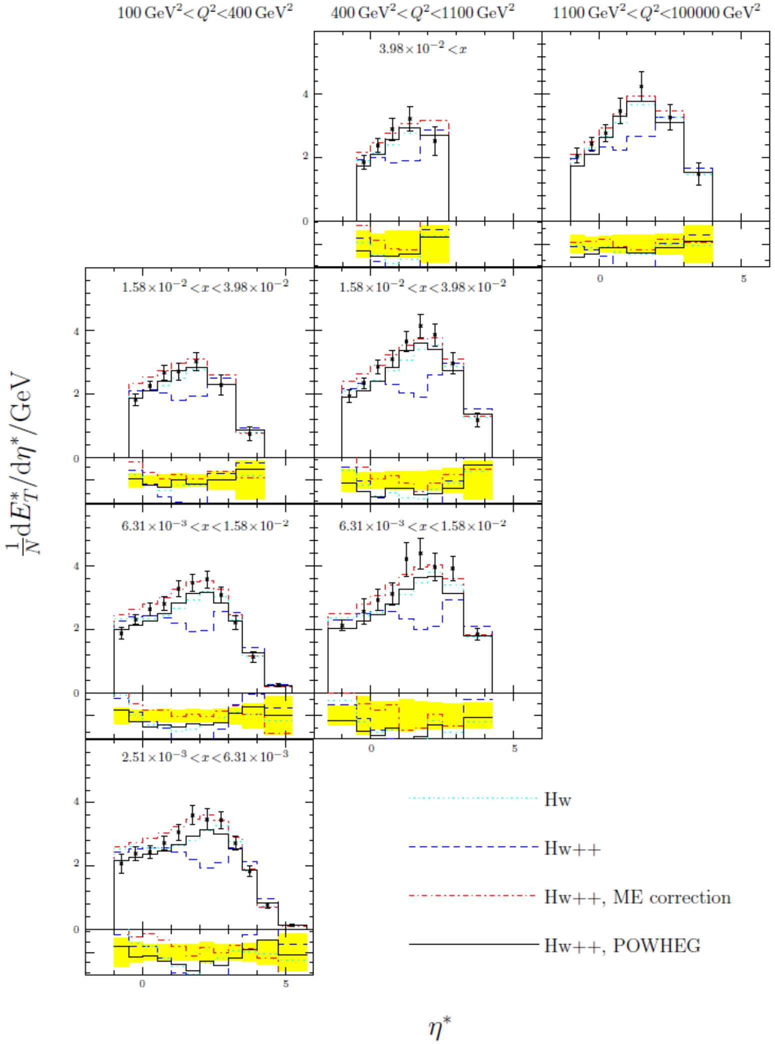
DIS Reduced Cross Section



PR and L d'Errico in preparation Herwig++ compared to ZEUS data Phys.Rev.D70:052001,2004, Eur.Phys.J.C28:175-201,2003

DIS Transverse Energy Flow

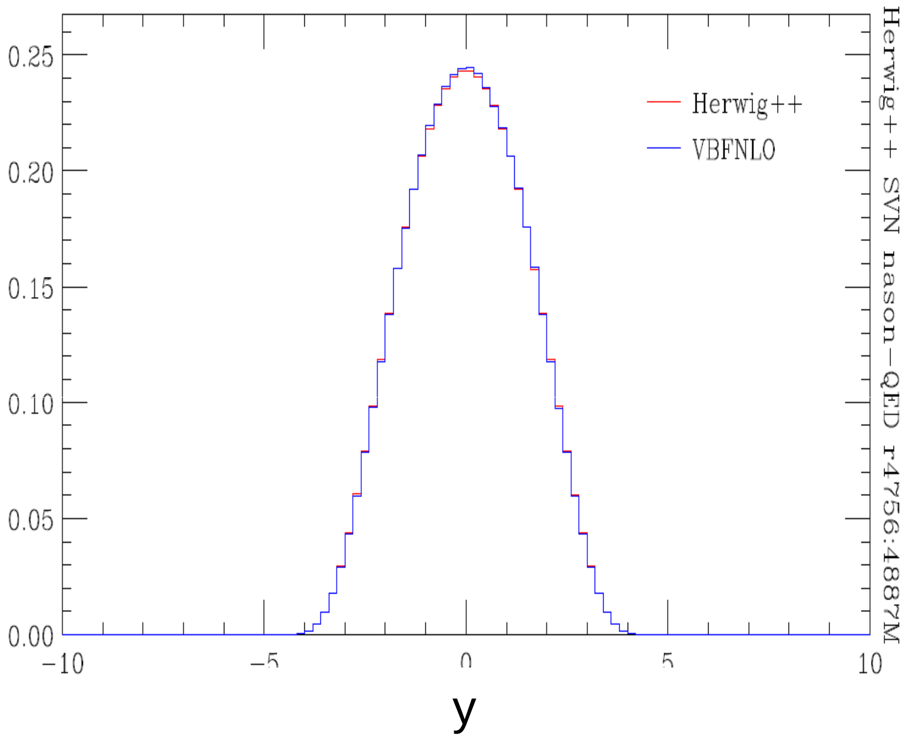




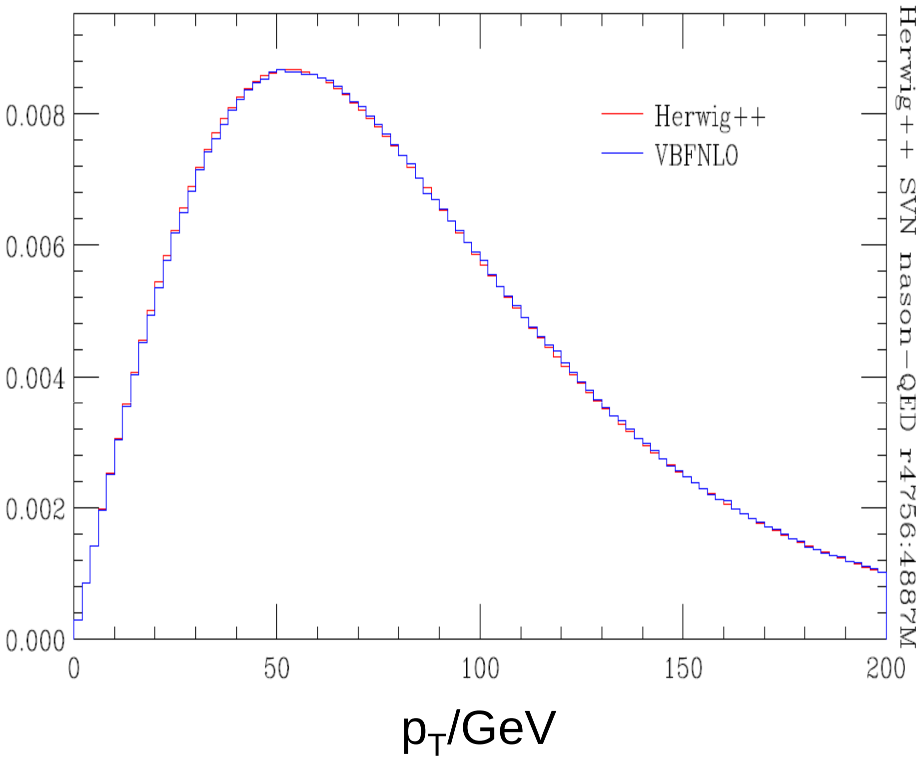
PR and L d'Errico in preparation Herwig++ compared to H1 data Eur.Phys.J.C12:595-607,2000.

VBF

Higgs Rapidity



Higgs p_T



PR and L d’Errico in preparation.

Multi-Jet Leading Order

- While the NLO approach is good for one hard additional jet and the overall normalization it cannot be used to give many jets.
- Therefore to simulate these processes use matching at leading order to get many hard emissions correct.
- I will briefly review the general idea behind this approach and then show some results.

CKKW Procedure

- Catani, Krauss, Kuhn and Webber JHEP 0111:063,2001.
- In order to match the ME and PS we need to separate the phase space:
  - one region contains the soft/collinear region and is filled by the PS;
  - the other is filled by the matrix element.
- In these approaches the phase space is separated using in k_T-type jet algorithm.

Durham Jet Algorithm

- For all final-state particles compute the resolution variables

$$d_{kB} \approx E_k^2 \theta_{kB}^2 \approx k_{\perp kB}^2 \qquad \theta_{kB}^2 \rightarrow 0$$

$$d_{kl} \approx \min(E_k^2, E_l^2)\theta_{kl}^2 \approx k_{\perp kl}^2 \qquad \theta_{kl}^2 \rightarrow 0$$

- The smallest of these is selected. If d_{kl} is the smallest the two particles are merged. If d_{kB} is the smallest the particle is merged with the beam.
- This procedure is repeated until the minimum value is above some stopping parameter d_{cut} .
- The remaining particles and pseudo-particles are then hard jets.

CKKW Procedure

- Radiation above a cut-off value of the jet measure is simulated by the matrix element and radiation below the cut-off by the parton shower.

1) Select the jet multiplicity with probability

$$P_n = \frac{\sigma_n}{\sum_{k=0}^{N} \sigma_k}$$

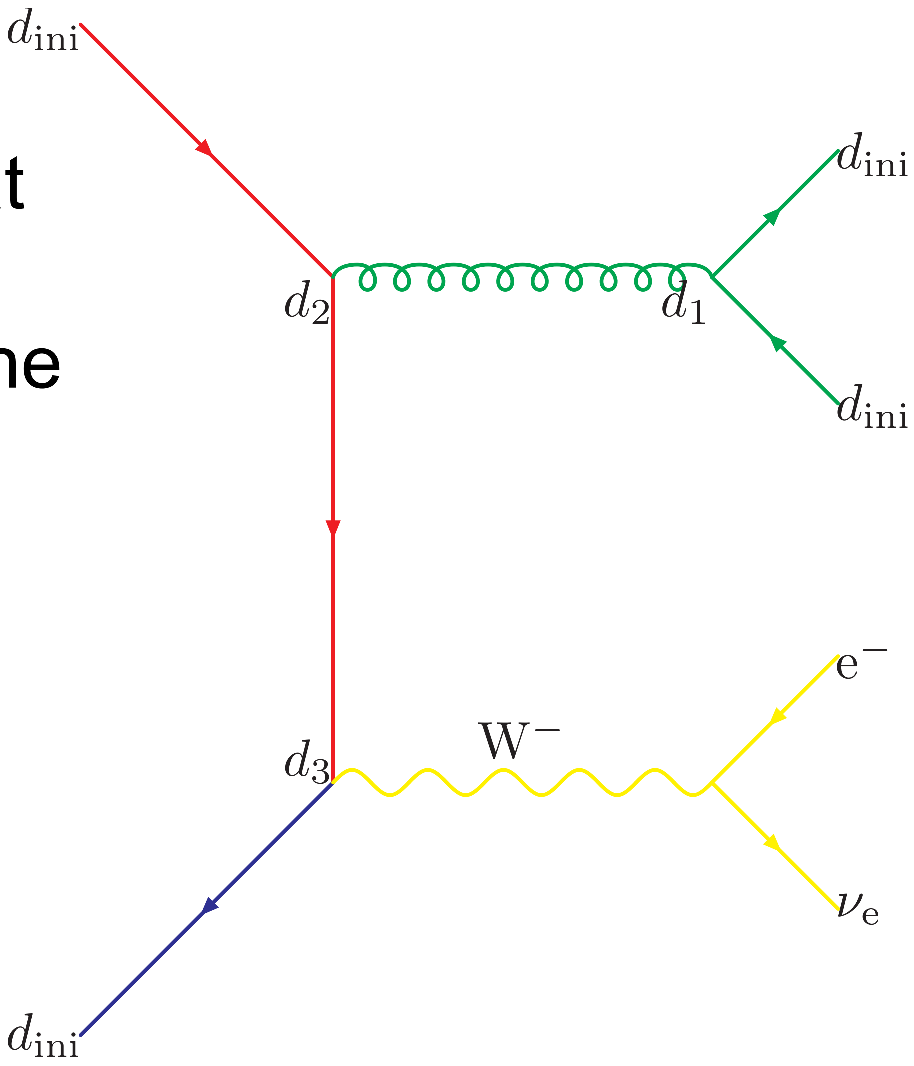
where σ_n is the *n*-jet matrix element evaluated at resolution d_{ini} using d_{ini} as the scale for the PDFs and α_S, *n* is the number of jets

2) Distribute the jet momenta according the ME.

CKKW Procedure

3) Cluster the partons to determine the values at which 1,2,..*n*-jets are resolved. These give the nodal scales for a tree diagram.
4) Apply a coupling constant reweighting.

$$\frac{\alpha_S(d_1)\alpha_S(d_2)\ldots\alpha_S(d_3)}{\alpha_S(d_{\text{ini}})^n} \leq 1$$

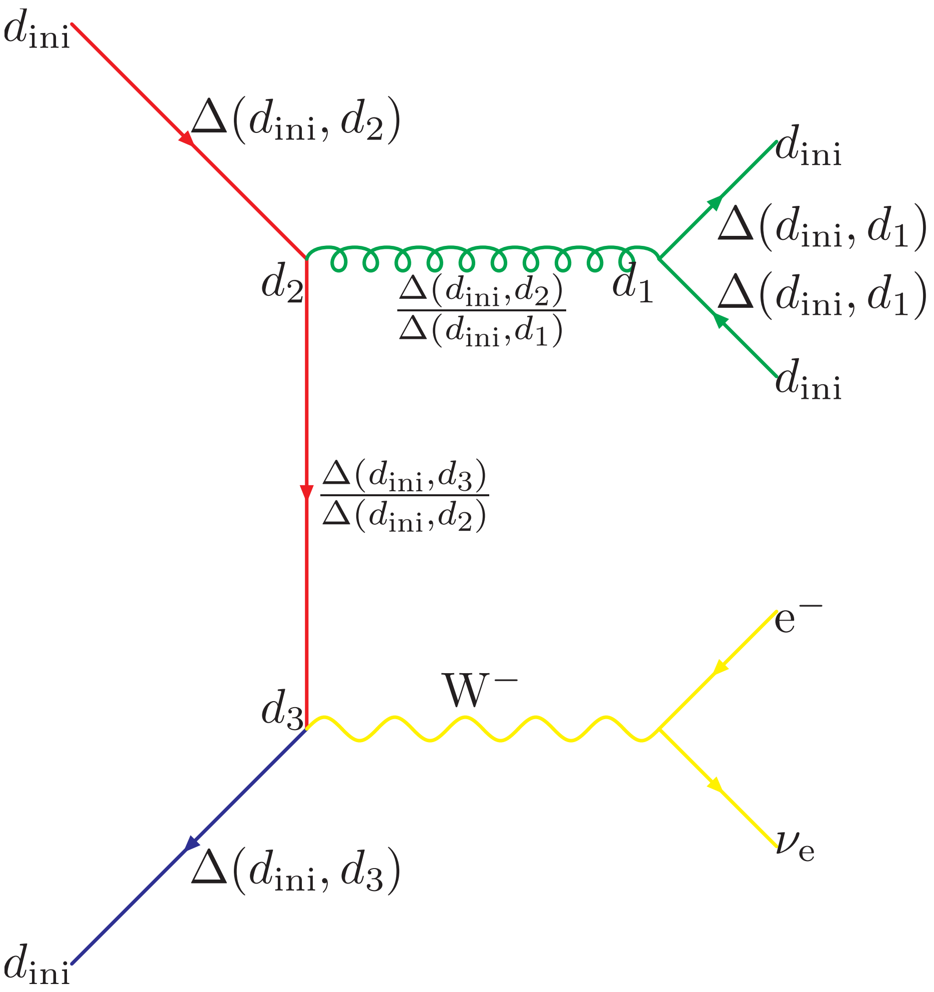


CKKW Procedure

5) Reweight the lines by a Sudakov factor

$$\frac{\Delta(d_{\text{ini}}, d_j)}{\Delta(d_{\text{ini}}, d_k)}$$

6) Accept the configuration if the product of the α_S and Sudakov weight is less than $R \in [0,1]$ otherwise return to step 1.

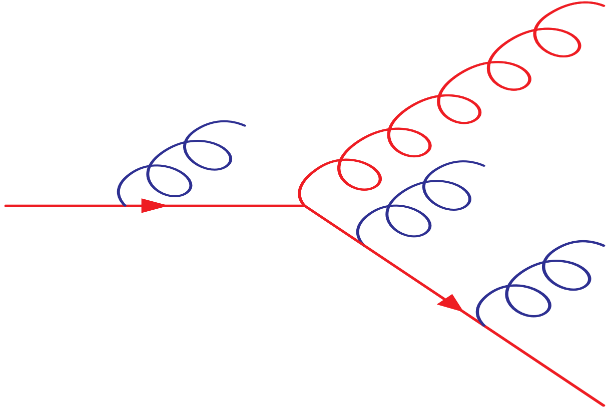


CKKW Procedure

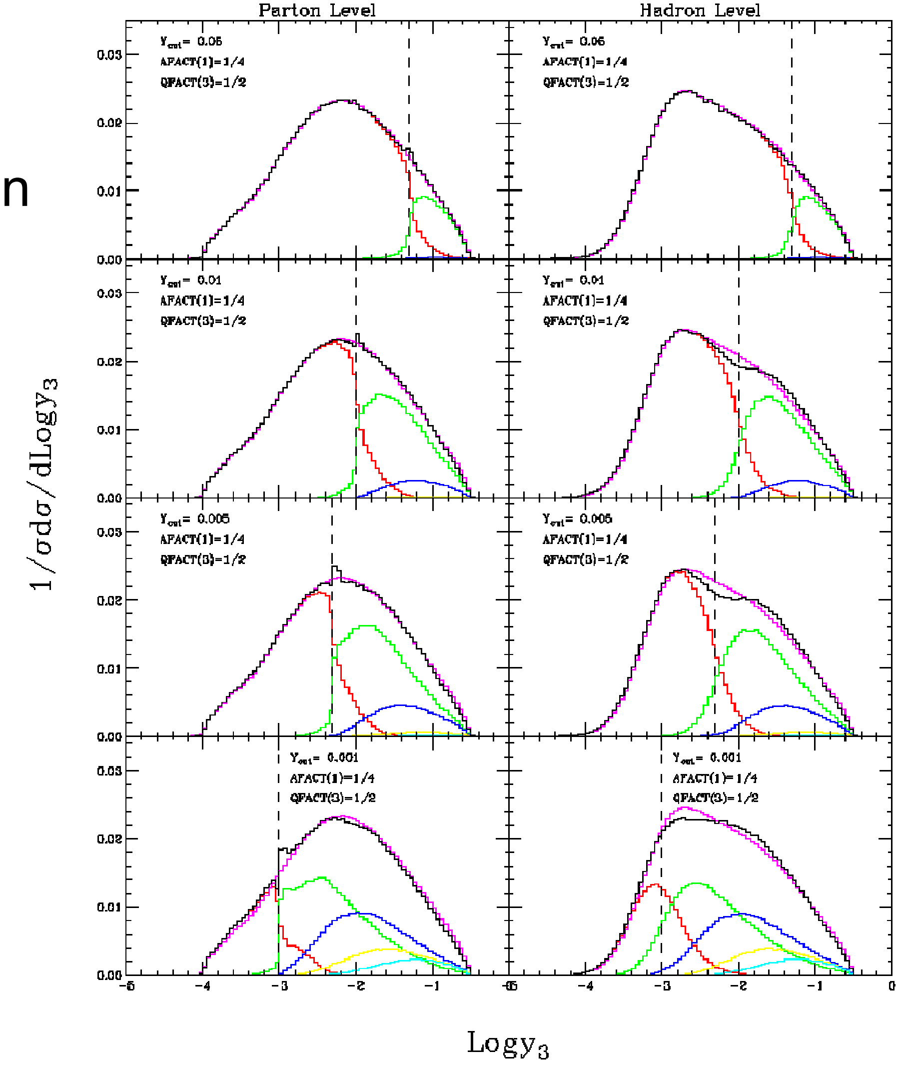
7) Generate the parton shower from the event starting the evolution of each parton at the scale at which it was created and vetoing emission above the scale d_{ini}.

Problems

- Enhanced starting scale for the evolution of the partons is designed to simulate soft, wide angle emission from the internal lines.
- CKKW gives the right amount of radiation



- But puts some of it in the wrong place with the wrong colour flow.

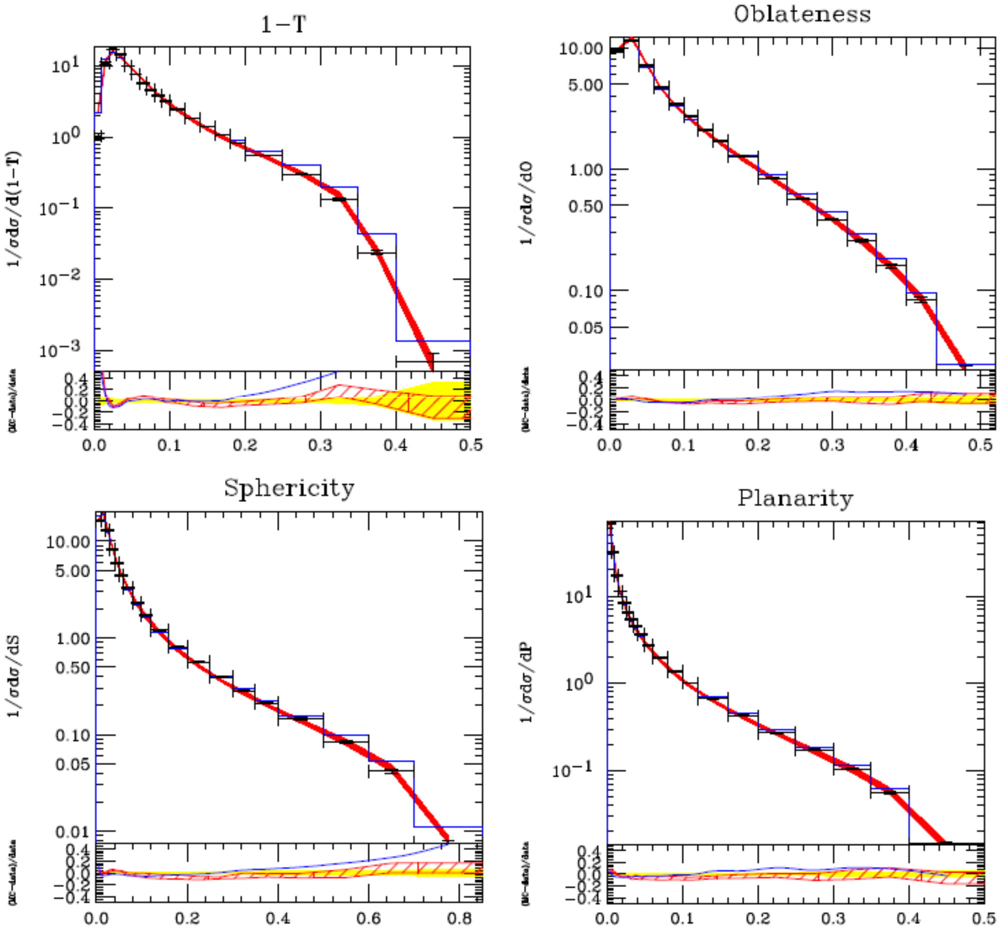


S. Mrenna and PR JHEP 0405: 04 (2004)

Solution

- The solution is that we should use a truncated shower to generate the soft wide angle emission.
- Nason's ideas can be generalised so that we replace step 7 of the CKKW procedure with
  - Map the momenta into a set of shower variables
  - Start the evolution as normal.
  - Evolve to the scale of the hardest emission and generate truncated showers from the internal lines.

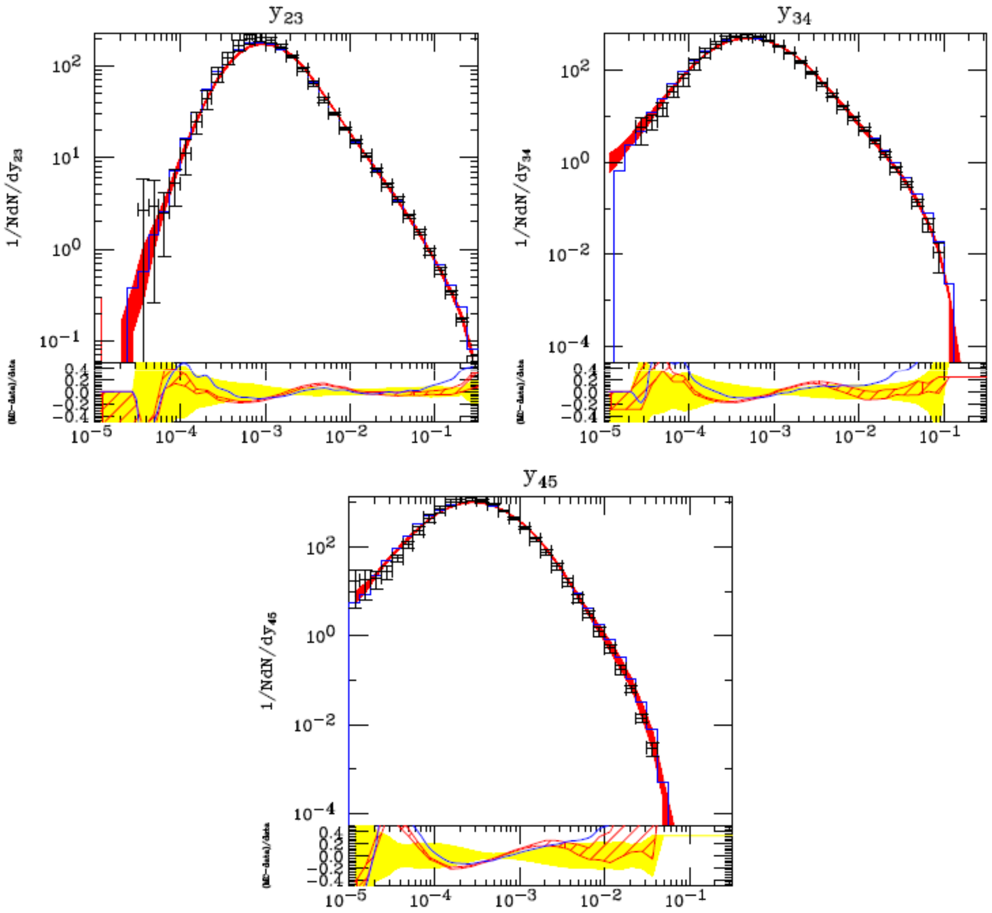
LEP Event Shapes



JHEP 0911:038,2009 Hamilton, PR, Tully

Herwig++ compared to DELPHI, Z.Phys.C73:11-60,1996

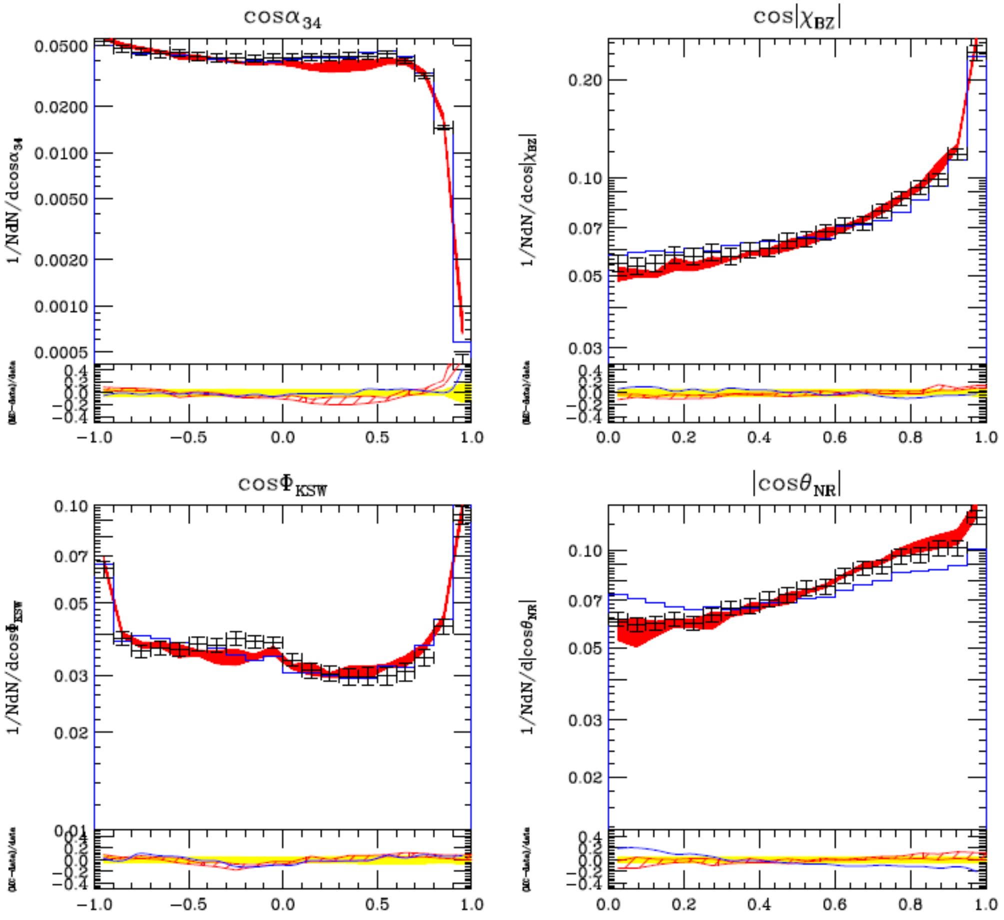
LEP Jet Distributions



JHEP 0911:038,2009 Hamilton, PR, Tully

Herwig++ compared to Eur.Phys.J.C17:19-51,2000

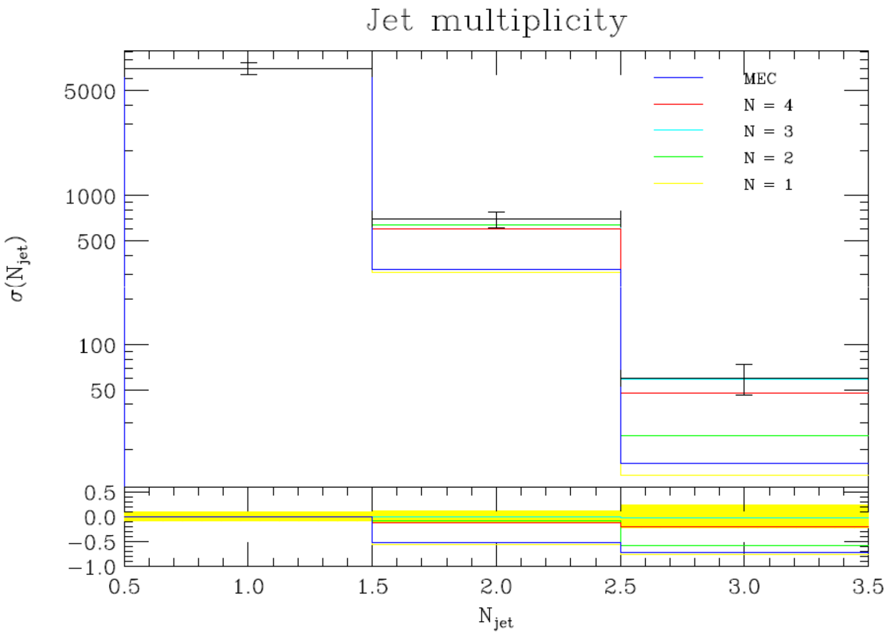
LEP Four Jet Angles

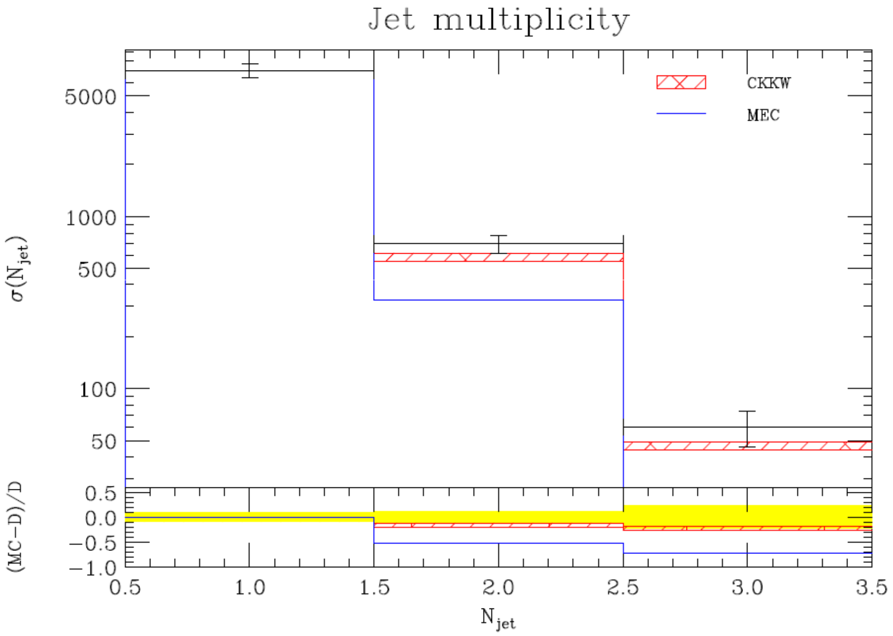


JHEP 0911:038,2009 Hamilton, PR, Tully

Herwig++ compared to Eur.Phys.J.C27:1-17,2003

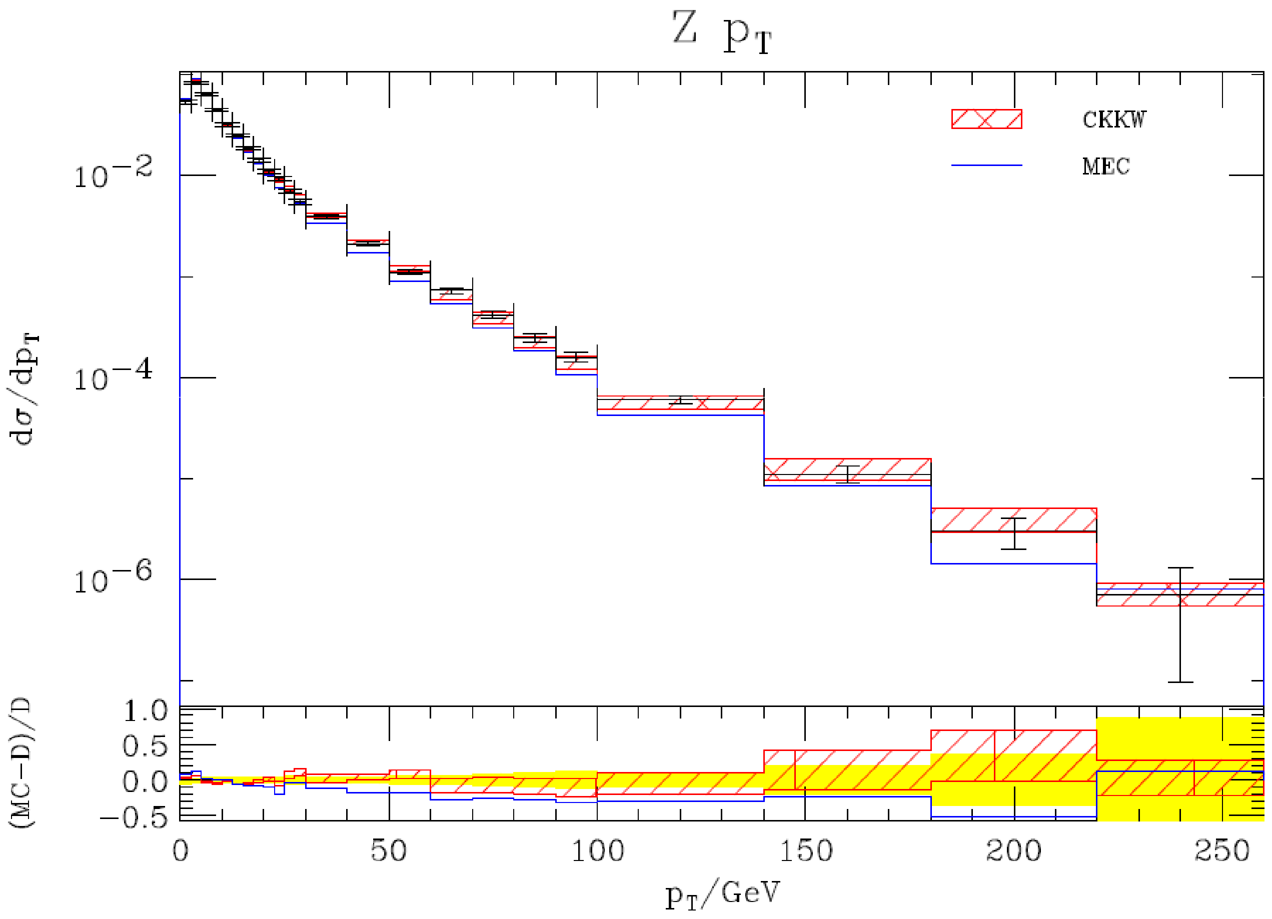
Jet Multiplicity in Z+jets at the Tevatron





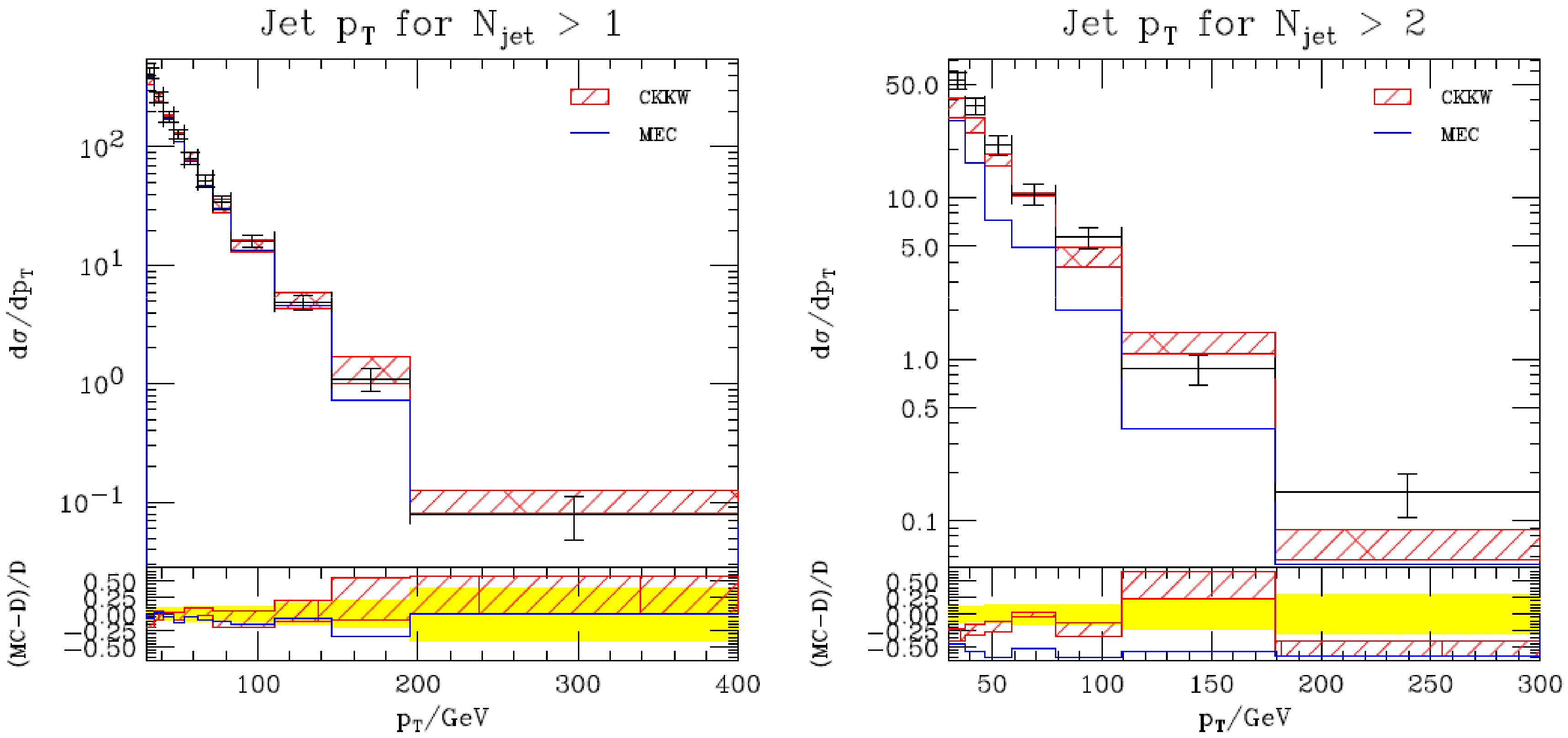
Herwig++ compared to data from CDF
Phys.Rev.Lett.100:102001,2008

p_T of the Z in Z+jets at the Tevatron



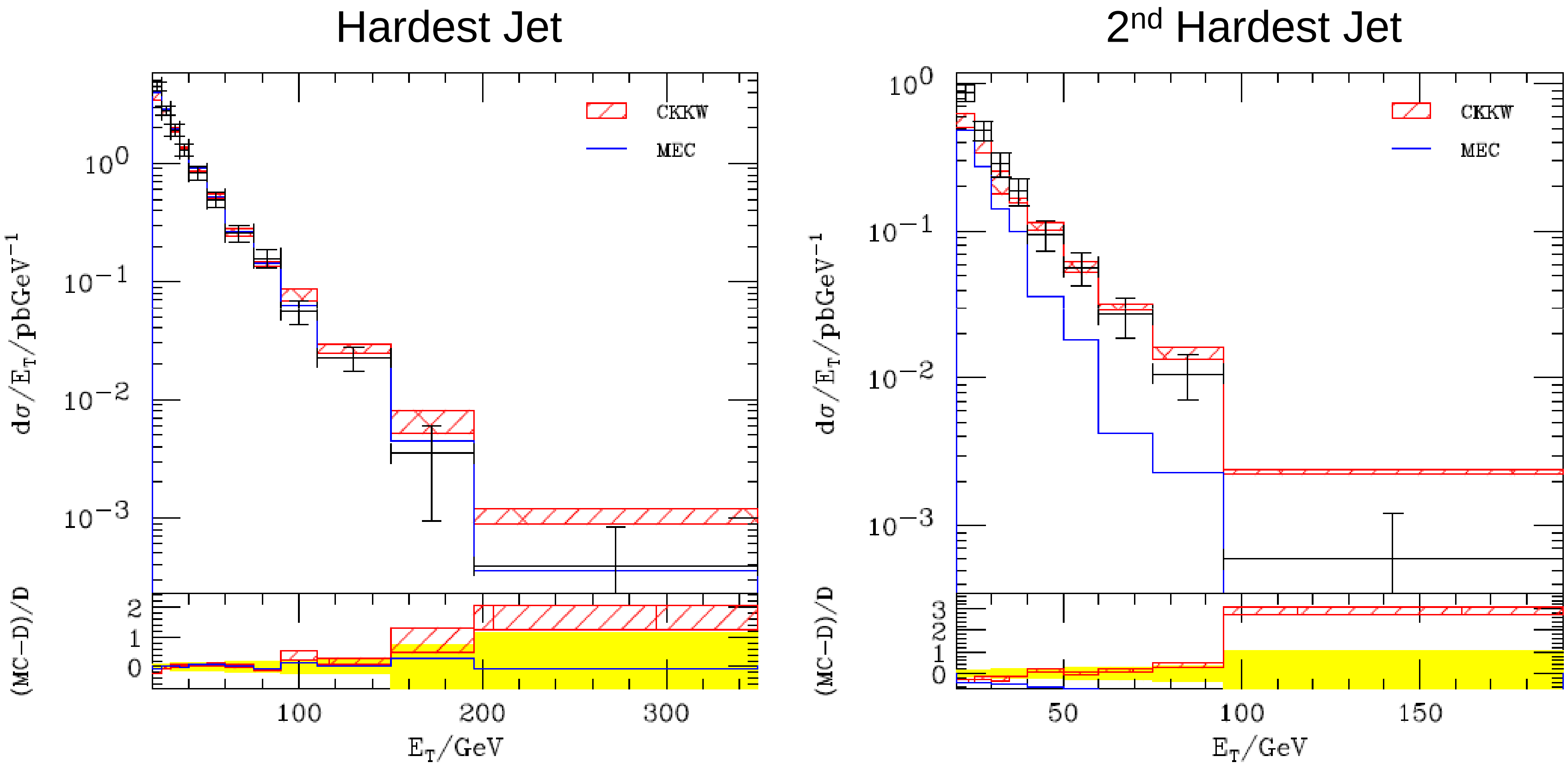
Herwig++ compared to data from D0
Phys.Rev.Lett.100:102002,2008

p_T of jets in Z+jets at the Tevatron



Herwig++ compared to data from CDF
Phys.Rev.Lett.100:102001,2008

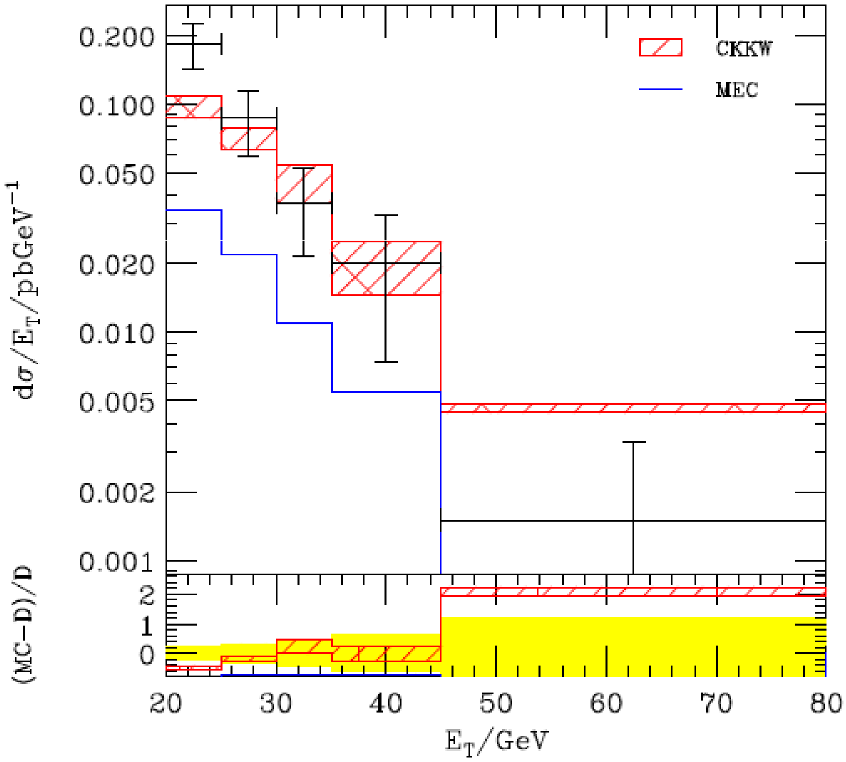
p_T of jets in W+jets at the Tevatron


Hardest Jet

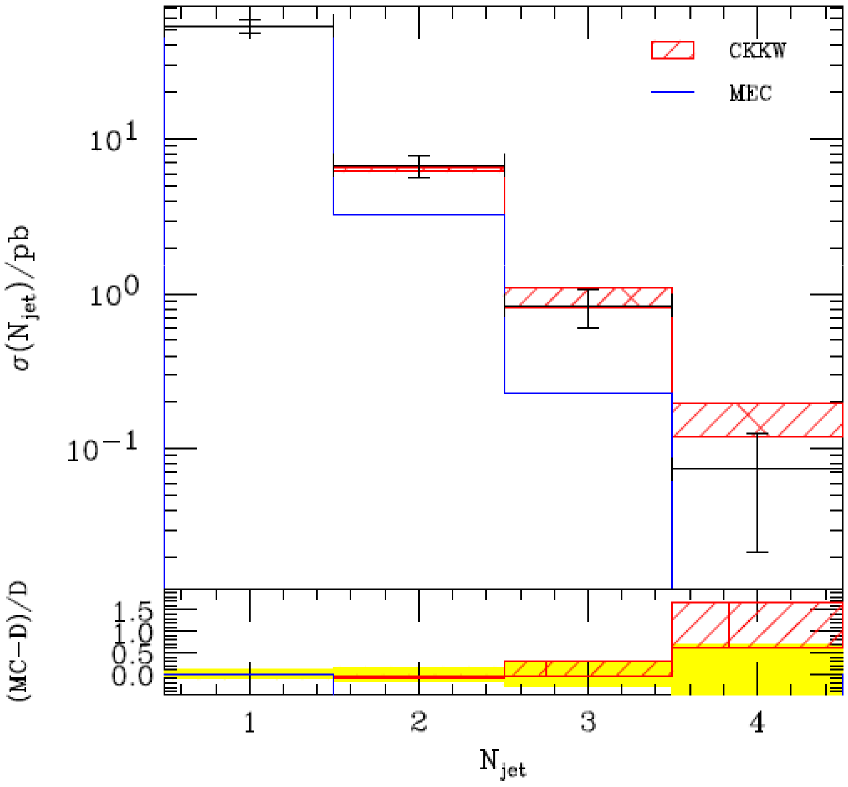
2nd Hardest Jet

Herwig++ compared to data from CDF
Phys.Rev.D77:011108,2008

p_T of jets in W+jets at the Tevatron



3rd Hardest Jet

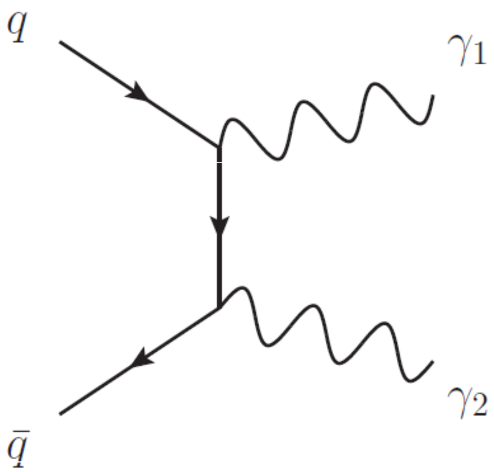


All Jets

Herwig++ compared to data from CDF Phys.Rev.D77:011108,2008

Diphoton Production

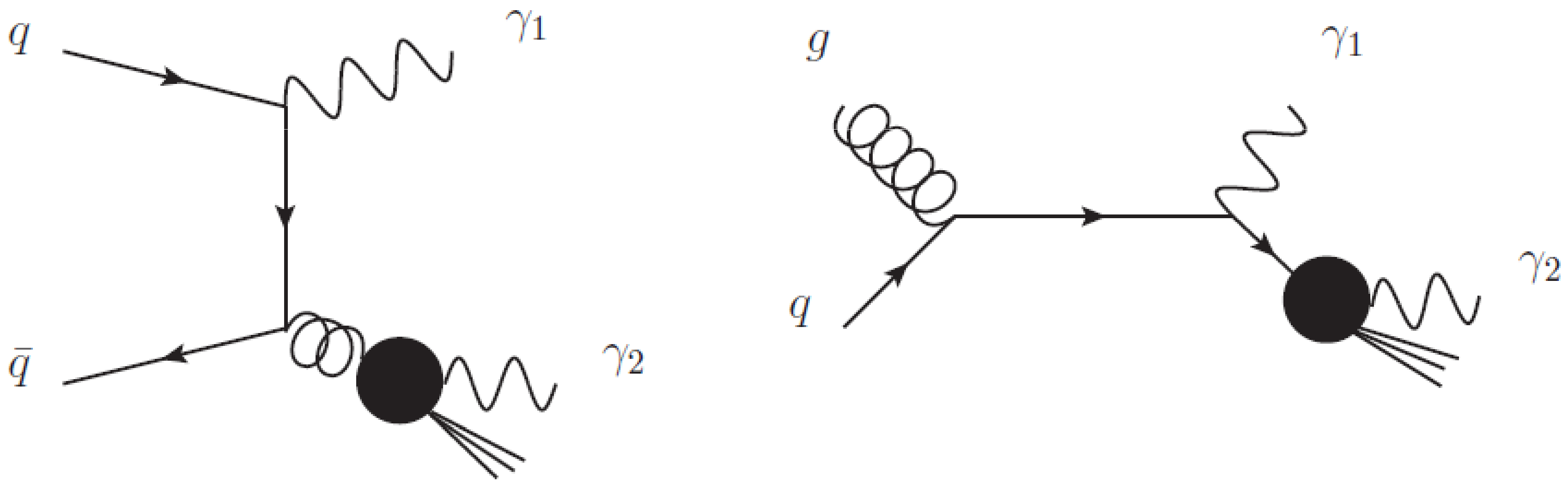
- Processes with photons in the final state look simple.



- However when calculating the higher order corrections and simulating them there are additional complications.

Diphoton Production

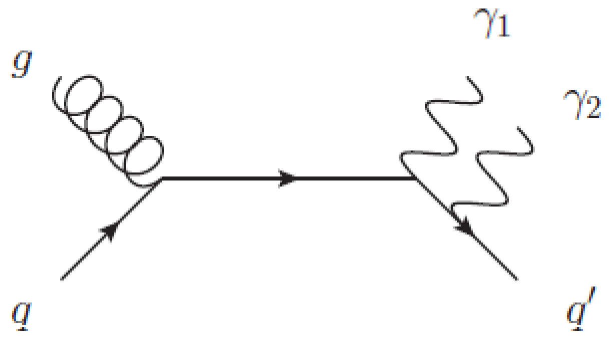
- In an analytic calculation need to include photon+jet production together with the photon fragmentation function.



- In a Monte Carlo simulation have to include photon+jet production and parton showering+hadronization.

Diphoton Production

- At NLO there is an additional problem.
- The real emission corrections include singularities when one of the photons is collinear with a final-state quark.

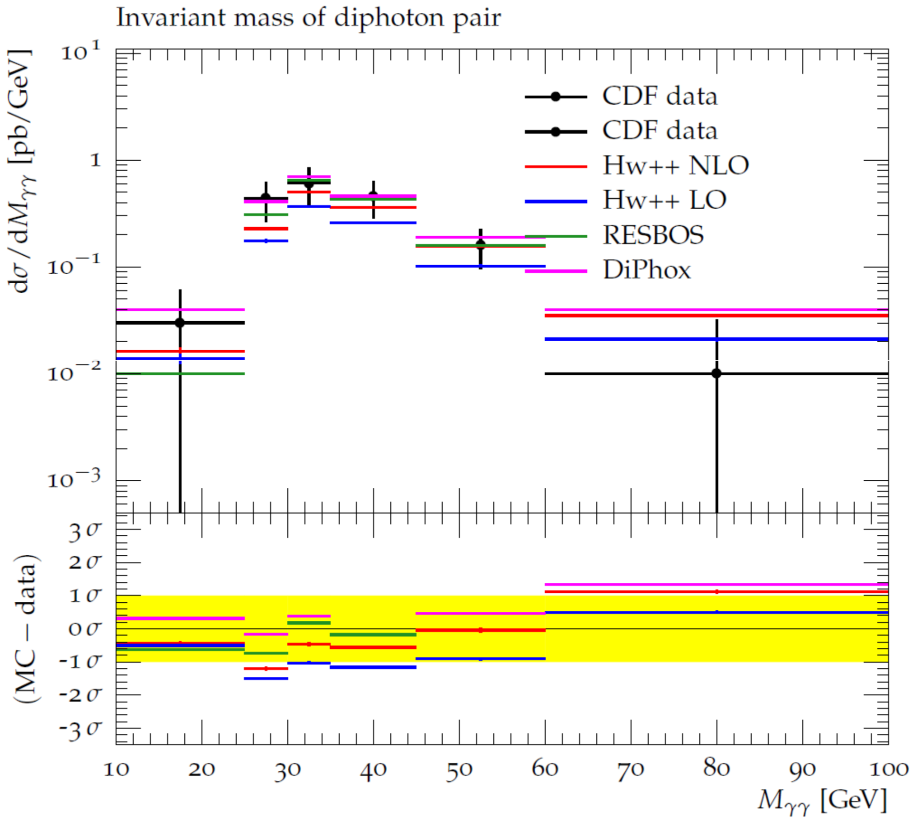


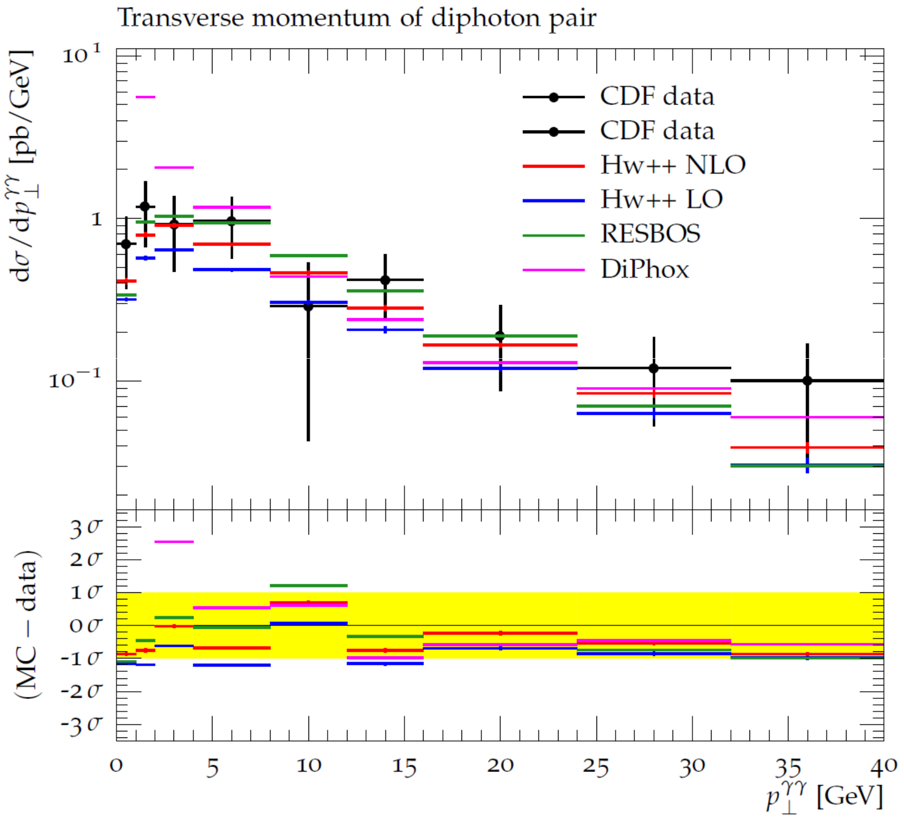
- Formally this has to be absorbed into the fragmentation function.
- Not clear how to proceed in an NLO simulation.

Diphoton Production

- Separate the real emission processes into:
  - a piece which is a QCD correction to diphoton production, contributes to both $\bar{B}$ and the Sudakov for hard emission;
  - a separate QED correction to photon+jet production, just contributes to a the Sudakov for hard QED emission.
- Allows NLO simulation with the shower still generating the photon fragmentation contribution.

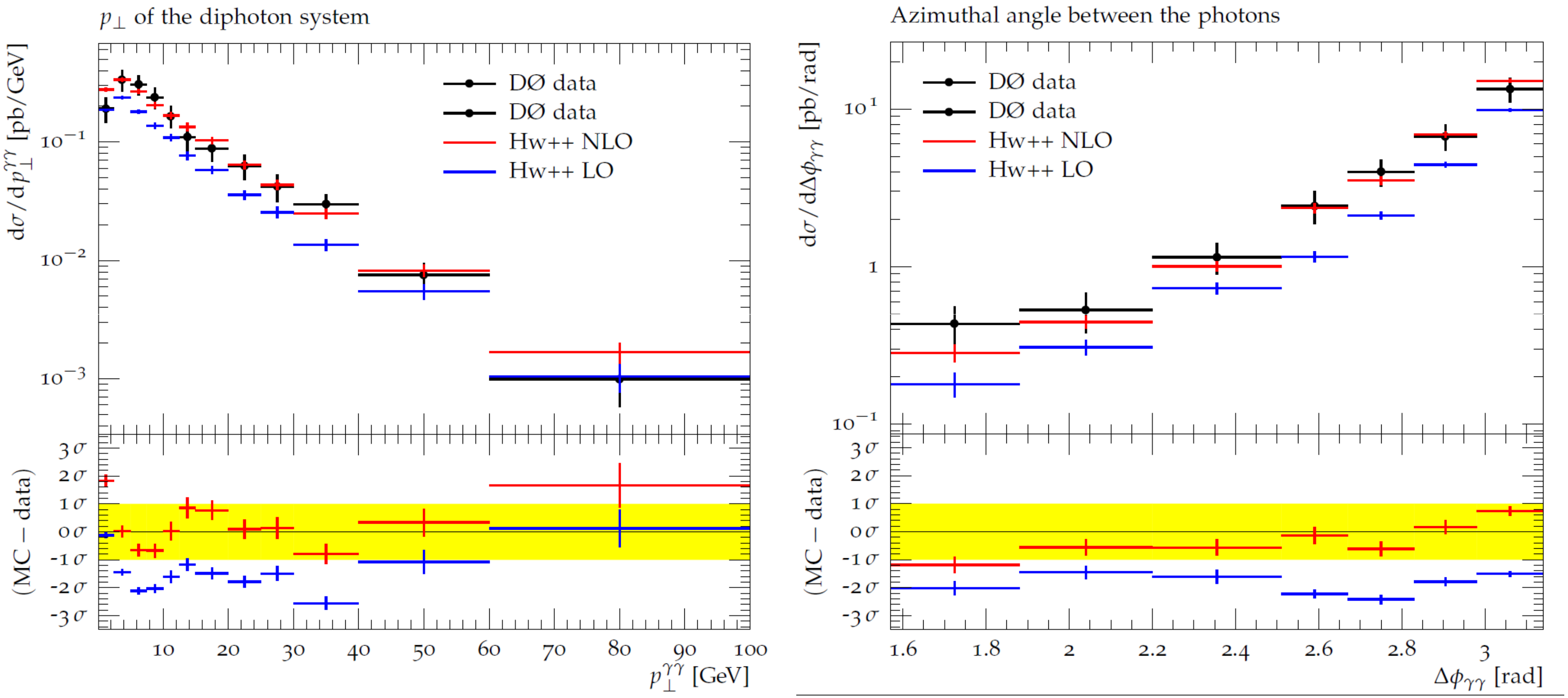
Diphoton Production





PR and L d’Errico in preparation Herwig++ compared to CDF data Phys. Rev. Lett. 95 022003, 2005.

Diphoton Production

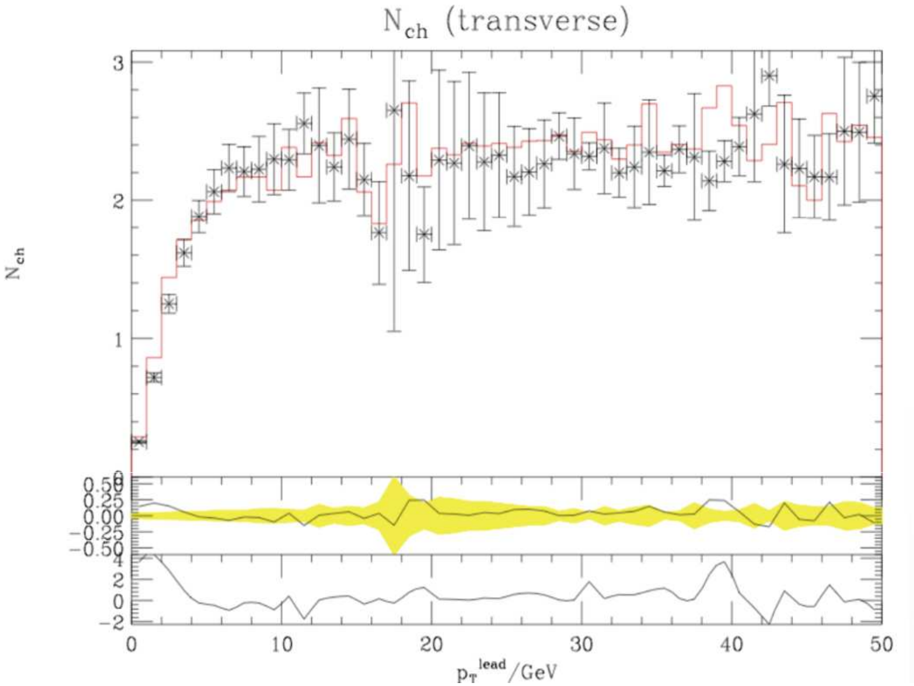


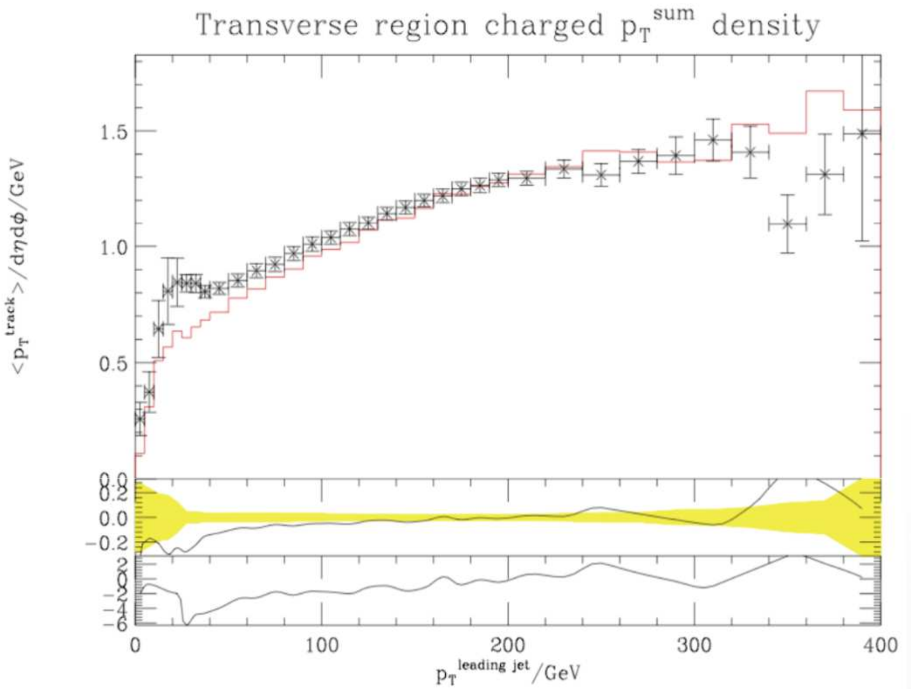
PR and L d’Errico in preparation Herwig++ compared to D0 data arXiv:1002.4917

Other things

- In addition to the improvements to the simulation of hard QCD radiation I’ve described Herwig++ includes many other improvements over the FORTRAN:
  - built in multiple parton interaction model for the underlying event and min bias;
  - full spin correlations in a wide range of BSM models, including the MSSM, MUED, the NMSSM will be available in the next release;
  - built in model of hadron and tau decays including spin correlations;
  - simulation of QED radiation in particle decays using the YFS approach.

Underlying Event

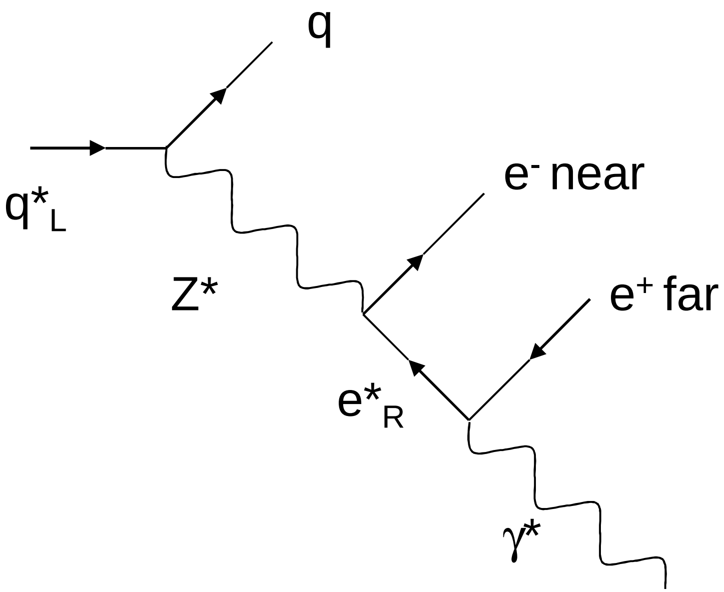


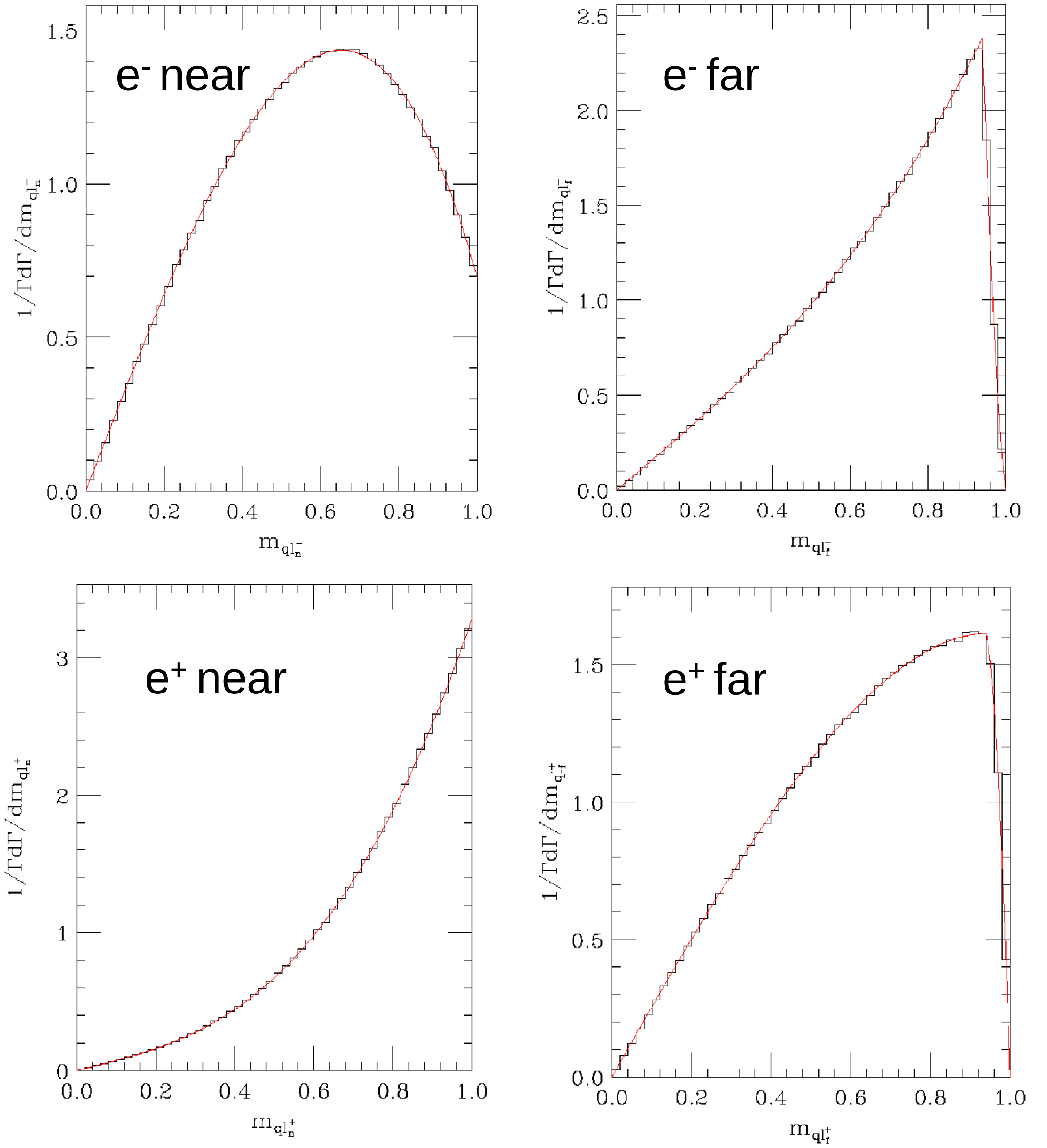


- Major new feature is a multiple scattering model of the underlying event.
- In good agreement with CDF data on the underlying event.

UED

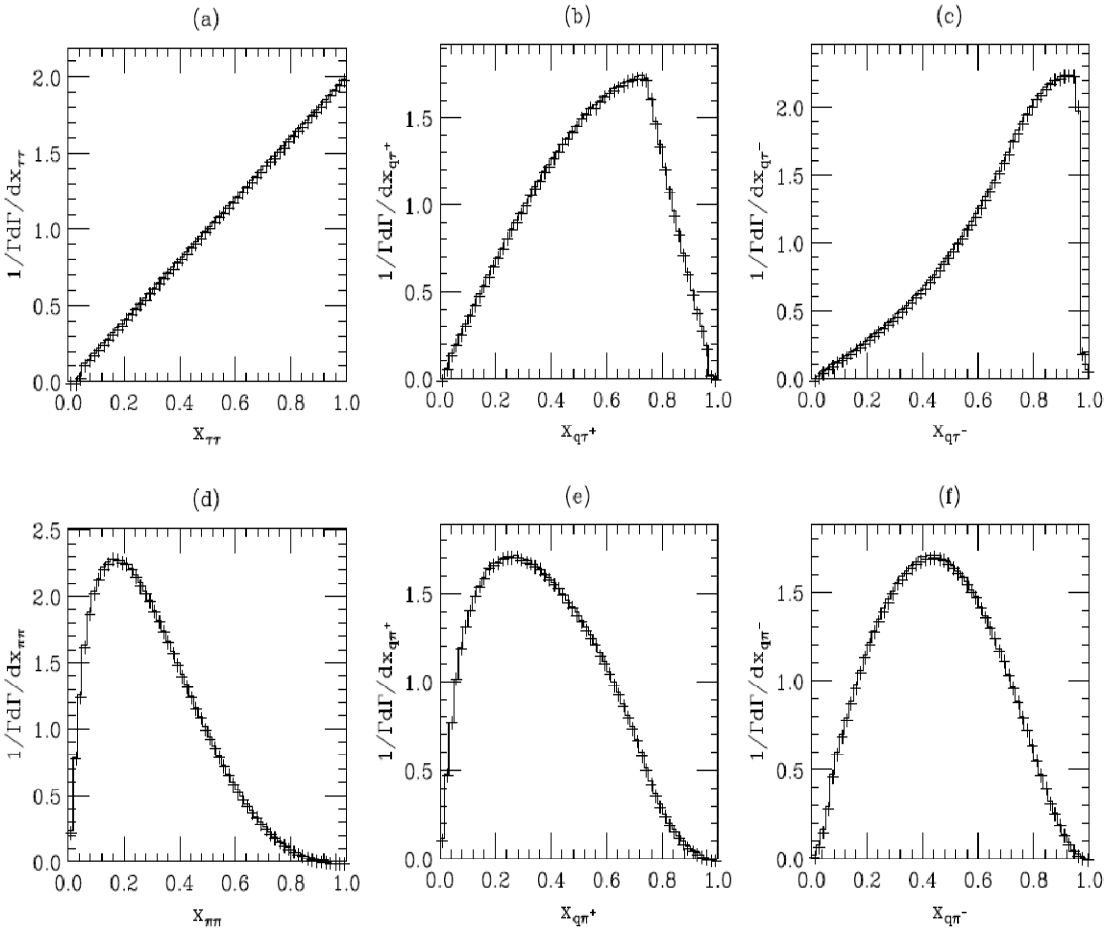
Look at the decay





Herwig++ compared to hep-ph/0507170 Smillie and Webber

Correlations in Tau Decays

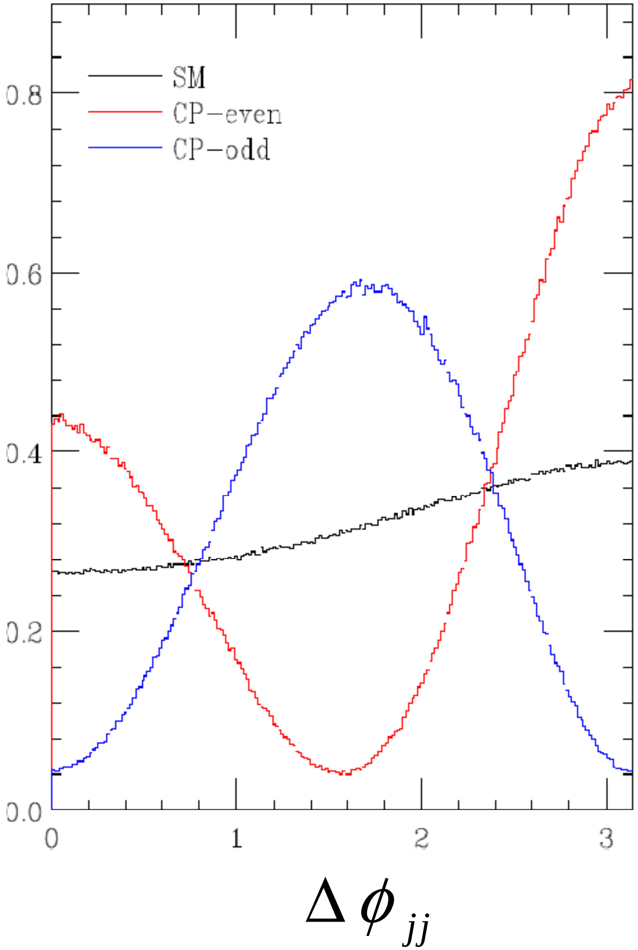


$$\tilde{q}_\alpha \to q\chi_2^0 \to \tau^\pm_{near}\tilde{\tau}_1^\mp \to \tau^\mp_{far}\chi_1^0$$

- Based on hep-ph/0612237 Choi et al.

VBF Higgs Production

- Much easier to make changes,
- To explore the CP structure of the Higgs can implement a new CP-even and CP-odd operators.
- Rest of the structure can then be used to calculate scattering processes and decays.



Based on hep-ph/0105325 Plehn et. al

Summary

- Herwig++ is now provides a sophisticated simulation of hadron collisions.
- The current version has NLO simulations of W and Z production, gg→H, W/Z+H, gauge boson pair production.
- The next release will include NLO simulations of DIS, VBF. As well as an improved CKKW approach for the simulation of many hard jets
- It’s taken a lot longer than we had expected.